Homework No. 01 (2024 Spring)

PHYS 510: CLASSICAL MECHANICS

School of Physics and Applied Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2024 Jan 30, 4.30pm

1. (**20 points.**) Consider the case when the friction force is quadratically proportional to velocity,

$$F_f = \frac{1}{2} D\rho A v^2, \tag{1}$$

where A is the area of crosssection of the object, ρ is the density of the medium, and D is a dimensionless drag coefficient. This should be contrasted with the case when the drag is linear in velocity. Typically, for small speeds, or when the size of the object is small, the drag force is linear in velocity. This is the case for motion in a highly viscous fluid, or for micron sized organisms in water. On the other hand, a sky diver, or a car on an interstate, experience quadratic drag forces.

(a) For a mass m falling under uniform gravity we have the equation of motion

$$m\frac{dv}{dt} = mg - F_f. \tag{2}$$

(b) Show that the terminal velocity, when $dv/dt = 0$, is given by

$$v_T = \sqrt{\frac{2mg}{D\rho A}}. \tag{3}$$

(c) Solve the equation of motion for the initial condition where the particle starts from rest, $v(0) = 0$, and show that it leads to the solution

$$v(t) = v_T \frac{\left(1 - e^{-\frac{2t}{\tau}}\right)}{\left(1 + e^{-\frac{2t}{\tau}}\right)}, \tag{4}$$

where $\tau = v_T/g$ sets the scale for time.

(d) The corresponding solution for linear drag is

$$v(t) = v_T\left(1 - e^{-\frac{t}{\tau}}\right), \tag{5}$$

where now $F_f = bv$ and $v_T = \frac{mg}{b}$ with $\tau = v_T/g$. Plot and compare the two velocity functions assuming the same terminal velocities.

2. (**20 points.**) Motion of a charged particle of mass m and charge q in a uniform magnetic field $\mathbf{B}$ and a uniform electric field $\mathbf{E}$ is governed by

$$m\frac{d\mathbf{v}}{dt} = q\,\mathbf{E} + q\,\mathbf{v}\times\mathbf{B}. \tag{6}$$

Choose $\mathbf{B}$ along the z-axis and $\mathbf{E}$ along the y-axis,

$$\mathbf{B} = 0\,\hat{\mathbf{i}} + 0\,\hat{\mathbf{j}} + B\,\hat{\mathbf{k}}, \tag{7a}$$
$$\mathbf{E} = 0\,\hat{\mathbf{i}} + E\,\hat{\mathbf{j}} + 0\,\hat{\mathbf{k}}. \tag{7b}$$

Solve this vector differential equation to determine the position $\mathbf{x}(t)$ and velocity $\mathbf{v}(t)$ of the particle as a function of time, for initial conditions

$$\mathbf{x}(0) = 0\,\hat{\mathbf{i}} + 0\,\hat{\mathbf{j}} + 0\,\hat{\mathbf{k}}, \tag{8a}$$
$$\mathbf{v}(0) = 0\,\hat{\mathbf{i}} + 0\,\hat{\mathbf{j}} + 0\,\hat{\mathbf{k}}. \tag{8b}$$

Verify that the solution is a cycloid characterized by the equations

$$x(t) = R(\omega_c t - \sin\omega_c t), \tag{9a}$$
$$y(t) = R(1 - \cos\omega_c t). \tag{9b}$$

where

$$R = \frac{E}{B\omega_c}, \qquad \omega_c = \frac{qB}{m}. \tag{10}$$

The particle moves as though it were a point on the rim of a wheel of radius R perfectly rolling (without sliding or slipping) with angular speed ω_c along the x-axis. It satisfies the equation of a circle of radius R whose center $(vt, R, 0)$ travels along the x-direction at constant speed v,

$$(x - vt)^2 + (y - R)^2 = R^2, \tag{11}$$

where $v = \omega_c R$.