Homework No. 11 (Spring 2024)

PHYS 205B: UNIVERSITY PHYSICS

School of Physics and Applied Physics, Southern Illinois University–Carbondale

Due date: Thursday, 2024 Apr 18, 4:00 PM, on D2L

Instructions

- You are encouraged to use any of the resources to complete this homework. However, the extent to which you depend on resources while doing homework is usually a measure of how much extra work you need to put in to master the associated concepts. Solutions should be the last resource.
- Describe your thought process in detail and organize it clearly. Make sure your answer has units and the right number of significant digits.
- After completion, scan the pages as a single PDF file, and submit the file on D2L (under Assessments → Assignments).

Problems

1. (**10 points.**) A solenoid has one hundred turns per 5.0 cm of it's length and has a crosssectional radius of 0.40 mm. It carries a current of 1.0 A.
   (a) What is the magnitude of the magnetic field generated by the solenoid?
   (b) What is the inductance per unit length of the solenoid.
   (c) How much magnetic energy is stored per unit length of the solenoid?

   Solution

2. (**10 points.**) Consider a coaxial cable of length l consisting of an inner conductor in the shape of a right circular cylinder of radius a surrounded by another concentric conductor in the shape of a cylindrical shell of radius b. Let a steady current flow in opposite directions in the inner of outer conductors.
   (a) Using Ampère's law show that the magnetic field generated by the two currents in the coaxial cable is
   $$B(\vec{\mathbf{r}}) = \begin{cases} 0, & r < a, \\ \hat{\boldsymbol{\phi}}\,\dfrac{\mu_0 I}{2\pi r}, & a < r < b, \\ 0, & b < r, \end{cases} \tag{1}$$
   so that the magnetic field, and the magnetic energy, generated by the wires is confined to the region between the conductors. Here r and ϕ are cylindrical polar coordinates on a plane perpendicular to the symmetry axis of the cable.

(b) Evaluate the magnetic flux passing through the half-plane $\phi = 0$ to be

$$\Phi_B = l\frac{\mu_0 I}{2\pi}\ln\frac{b}{a}. \tag{2}$$

(c) Determine the inductance of the coaxial cable, using $L = \Phi_B/I$, to be

$$L = l\frac{\mu_0}{2\pi}\ln\frac{b}{a}, \tag{3}$$

which depends only on the geometry of the cable. Calculate the inductance per unit length of a coaxial cable with inner radius of 2.0 mm and outer radius of 4.0 mm.

Solution

3. (**10 points.**) Consider a series RL circuit.

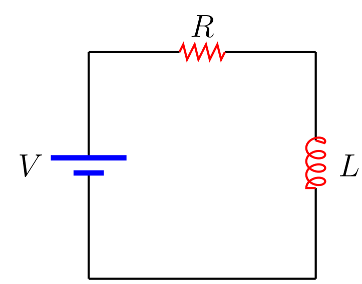

Figure 1: A series RL circuit.

(a) Using Kirchhoff's law, write down the (differential) equation relating the current I in the circuit and the rate of change of current dI/dt in the circuit.

(b) The solution to the equation, in Question (3a), for the initial condition $I(0) = 0$ is given by

$$I(t) = \frac{V}{R}\left[1 - e^{-\frac{t}{(L/R)}}\right]. \tag{4}$$

What is the maximum current that flows through the circuit? How much time does it take to attain this maximum current in the circuit? How much time does it take to attain half of this maximum current in the circuit? Given $L = 1.0\,\mathrm{mH}$, $R = 1.0\,\mathrm{M\Omega}$, and $V = 110.0\,\mathrm{V}$.

Solution

4. (**10 points.**) Consider a LC circuit.

(a) Using Kirchhoff's law write the differential equation governing an inductor and a capacitor in series.

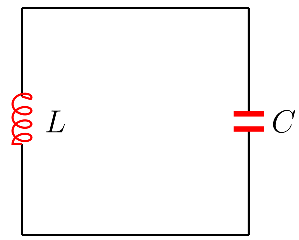

Figure 2: LC circuit.

(b) For the initial conditions

$$Q(0) = Q_0, \tag{5a}$$
$$I(0) = 0, \tag{5b}$$

the solution to the differential equation in Question (4a) is

$$Q(t) = Q_0 \cos \omega t \tag{6}$$

and

$$I(t) = -\omega Q_0 \sin \omega t, \tag{7}$$

where the angular frequency of oscillation is given by

$$\omega = \frac{1}{\sqrt{LC}}. \tag{8}$$

(c) A simple radio receiver uses a LC circuit to tune. If such a radio uses a variable capacitor and a 1.0 mH inductor, what is the capacitance that will tune to a 5.0 MHz signal?

Solution

5. (**10 points.**) Electromagnetic waves are oscillations of electric and magnetic fields that sustain each other using Faraday and Maxwell laws. The speed of all electromagnetic waves is the same in vacuum and is called the speed of light in vacuum and is given by

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}. \tag{9}$$

In SI units c is chosen to be a whole number,

$$c = 299\,792\,458 \, \frac{\text{m}}{\text{s}}. \tag{10}$$

(a) The wave nature stipulates the relation between wavelength λ, frequency f, and speed c of the wave,

$$c = \lambda f. \tag{11}$$

The time period $T = 1/f$, and the wavevector $k = 2\pi/\lambda$, are related quantities. Calculate the frequency associated with a monochromatic wave of red light of wavelength 632.8 nm.

(b) The electromagnetic energy density is given by

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2. \tag{12}$$

Given that the red monochromatic wave consists of a maximum electric field strength of 20.0 V/m, determine the associated maximum magnetic field strength. (Hint: $E = cB$.) Calculate the electromagnetic energy per unit volume for the red light. Also, show that the electrical energy density is equal to the magnetic energy density.

(c) The flux of the electromagnetic energy density, a measure of the flow rate of electromagnetic energy per unit area, is given by the Poynting vector

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}. \tag{13}$$

The electromagnetic momentum density is given by

$$\vec{\mathbf{G}} = \frac{1}{c^2}\vec{\mathbf{S}}. \tag{14}$$

Calculate the magnitude of the electromagnetic momentum density for the red light.

Solution