

## Homework No. 10 (Spring 2024)

### PHYS 205B: UNIVERSITY PHYSICS

*School of Physics and Applied Physics, Southern Illinois University–Carbondale*

Due date: Thursday, 2024 Apr 11, 4:00 PM, on D2L

### Instructions

- You are encouraged to use any of the resources to complete this homework. However, the extent to which you depend on resources while doing homework is usually a measure of how much extra work you need to put in to master the associated concepts. Solutions should be the last resource.
- Describe your thought process in detail and organize it clearly. Make sure your answer has units and the right number of significant digits.
- After completion, scan the pages as a single PDF file, and submit the file on D2L (under Assessments → Assignments).

### Problems

1. (**10 points.**) An elastic conducting material is stretched into a circular loop of 15 cm radius. It is placed with its plane perpendicular to a uniform 0.50 T magnetic field. When released, the radius of the loop starts to shrink at the rate of 72 cm/s.
  - (a) What is the direction of the induced current?
  - (b) What EMF is induced in the loop?

#### Solution

2. (**10 points.**) Figure 1 shows a conducting rod being pulled along horizontal, frictionless, conducting rails at a constant speed  $v$ . A uniform magnetic field  $\mathbf{B}$  fills the region in which the rod moves. Assume  $L = 10.0$  cm,  $v = 5.0$  m/s,  $B = 1.2$  T, and  $R = 0.40$   $\Omega$ .
  - (a) Is the magnetic flux in the loop increasing or decreasing?
  - (b) What is the direction of the induced current in the loop?
  - (c) Determine the magnitude of the induced current in the loop.

#### Solution

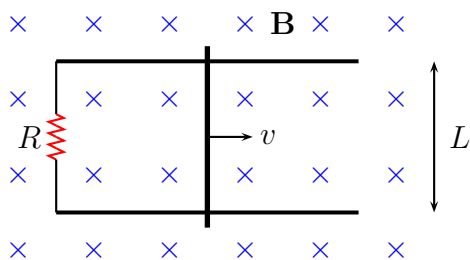


Figure 1: Problem 2

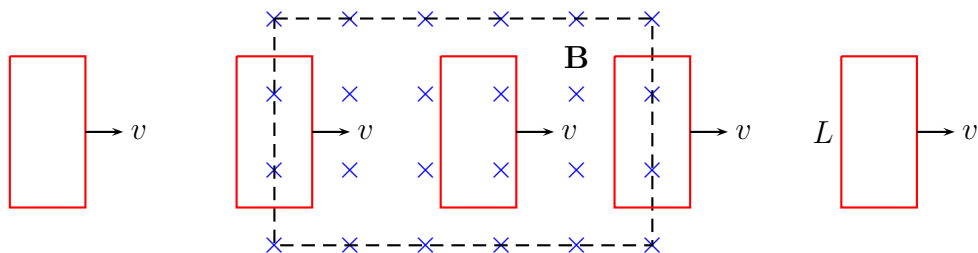


Figure 2: Problem 3.

3. (10 points.) Figure 2 shows five snapshots of a rectangular coil being pushed across the dotted region where there is a uniform magnetic field directed into the page. Outside of this region the magnetic field is zero. Determine the direction of induced current in the loop at each of the five instances in the figure. Given  $L = 10.0$  cm,  $v = 5.0$  m/s, and  $B = 1.2$  T, determine the induced EMF in the loop at each of the five instances in the figure.

**Solution**

4. (10 points.) Two parallel rails with negligible resistance are a distance  $L = 12$  cm apart and are connected by a resistor of resistance  $R = 5.00 \Omega$ . The circuit also contains two metal rods having negligible resistances sliding along the rails, see Fig. 3. The rods are pulled at constant speeds of  $v_1 = 4.00$  m/s and  $v_2 = 2.00$  m/s, respectively. A uniform magnetic field of magnitude  $B = 0.010$  T is applied perpendicular to the plane of the rails. Determine the direction and magnitude of current in resistance  $R_3$ . Let  $R_1 = R_2 = R_3 = R$ .

[Solution (Erratum: At timestamp 1:12:33 it should be  $I_1 = 0.5$  mA, which leads to  $I_2 = 0$  and  $I_3 = 0.5$  mA.)]

5. (10 points.) A transformer consists of two coils wound on the same cylinder such that the flux through both the coils is the same, that is,

$$\frac{d\Phi_1}{dt} = \frac{d\Phi_2}{dt}. \quad (1)$$

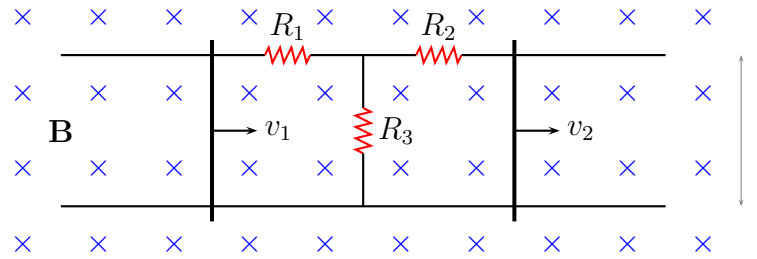


Figure 3: Problem 4

Thus, using Faraday's law derive the ratio of the voltages in the two coils to be given by

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}. \quad (2)$$

Energy conservation requires the power in the coils to be the same, that is  $P_1 = P_2$ . Thus, further derive

$$\frac{I_2}{I_1} = \frac{N_1}{N_2}. \quad (3)$$

A device operates at  $V_2 = 10.0 \text{ V}$ . It uses a transformer to get the required voltage when plugged into a wall socket with voltage  $V_1 = 120 \text{ V}$ .

- (a) Determine the ratio of the turns in the two coils inside the transformer.
- (b) If the device pulls a current of 120 mA, determine the current coming out of the wall socket.

### Solution

6. (10 points.) Consider the area enclosed by the loop formed in the configuration shown in Figure 4. The rotation described in the figure effectively changes the area enclosed by the loop periodically.

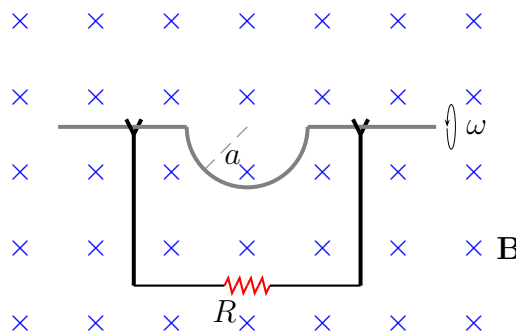


Figure 4: Problem 6

(a) For uniform angular speed of rotation  $\omega$ , with  $\theta = \omega t$ , show that

$$\frac{d \cos \theta}{dt} = -\omega \sin \omega t, \quad (4)$$

Then, show that the induced EMF in the loop is given by

$$\Delta V_{\text{eff}} = BA\omega \sin \omega t. \quad (5)$$

Determine the maximum induced voltage for  $B = 0.10 \text{ T}$ , radius  $a = 10.0 \text{ cm}$ , and angular speed of rotation of 600 revolutions per minute ( $\omega = 20.0\pi \text{ rad/s}$ ).

(b) Qualitatively plot the induced voltage as a function of time.

**Solution**