

## Homework No. 07 (Spring 2024)

### PHYS 205B: UNIVERSITY PHYSICS

*School of Physics and Applied Physics, Southern Illinois University–Carbondale*

Due date: Thursday, 2024 Mar 21, 4:00 PM, on D2L

### Instructions

- You are encouraged to use any of the resources to complete this homework. However, the extent to which you depend on resources while doing homework is usually a measure of how much extra work you need to put in to master the associated concepts. Solutions should be the last resource.
- Describe your thought process in detail and organize it clearly. Make sure your answer has units and the right number of significant digits.
- After completion, scan the pages as a single PDF file, and submit the file on D2L (under Assessments → Assignments).

### Problems

1. (**10 points.**) An electron that has velocity  $\vec{v} = (2.1 \times 10^6 \text{ m/s}) \hat{\mathbf{i}} + (2.7 \times 10^6 \text{ m/s}) \hat{\mathbf{j}}$  moves through a magnetic field  $\vec{\mathbf{B}} = (0.03 \text{ T}) \hat{\mathbf{i}} - (0.15 \text{ T}) \hat{\mathbf{j}}$ . Find the force on the electron.

#### Solution

2. (**10 points.**) Motion of a charged particle of mass  $m$  and charge  $q$  in a uniform magnetic field  $\mathbf{B}$  is governed by the Lorentz equation

$$m \frac{d\mathbf{v}}{dt} = q \mathbf{v} \times \mathbf{B}. \quad (1)$$

For the case when  $\mathbf{B}$  is pointing along the  $z$ -axis, show that the above vector equation corresponds to the following three coupled differential equations,

$$\frac{dv_x}{dt} = \omega v_y, \quad (2a)$$

$$\frac{dv_y}{dt} = -\omega v_x, \quad (2b)$$

$$\frac{dv_z}{dt} = 0, \quad (2c)$$

for the components of velocity of the particle. (Rest of this problem need not be submitted for assessment.) These differential equations can be solved to determine the position  $\mathbf{x}(t)$  and velocity  $\mathbf{v}(t)$  of the particle as a function of time. In particular, for initial conditions,

$$\mathbf{x}(0) = 0\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 0\hat{\mathbf{k}}, \quad (3a)$$

$$\mathbf{v}(0) = 0\hat{\mathbf{i}} + v_0\hat{\mathbf{j}} + 0\hat{\mathbf{k}}, \quad (3b)$$

it can be shown that the solution describes a circle of radius  $R = mv_0/(qB)$  centered at position  $R\hat{\mathbf{i}}$ .

### Solution

3. (10 points.) A proton and an electron enters a region containing a magnetic field going into the page,  $\vec{\mathbf{B}} = -2.0\hat{\mathbf{k}}\text{ T}$ . Let the velocity of both the particles while they enter the region be to the right,  $\vec{\mathbf{v}} = 3.0 \times 10^5\hat{\mathbf{i}}\text{ m/s}$ .
- Determine the magnitude of the magnetic force on the proton and the electron.
  - Determine the direction of the magnetic force on the proton and the electron, using the right-hand rule.
  - Determine the corresponding accelerations experienced by the proton and the electron.
  - Determine the cyclotron frequency of the proton and the electron.
  - Determine the radius of the circle described by the paths of the proton and the electron.

**Solution** (Erratum: At Timestamp 10:30 the radius  $R$  should be  $8.6 \times 10^{-7}\text{ m}$ .)

4. (10 points.) The electric field and the magnetic field both deflect charged particles due to the respective forces. In a velocity selector these forces are exactly balanced for particles moving with a particular velocity which go through undeviated. See Figure 1. Determine the magnitude and direction of the velocity selected by a velocity selector consisting of an electric field of  $\mathbf{E} = -3.0 \times 10^3\hat{\mathbf{j}}\text{ N/C}$  and a magnetic field of  $\mathbf{B} = -1.5\hat{\mathbf{k}}\text{ T}$ .

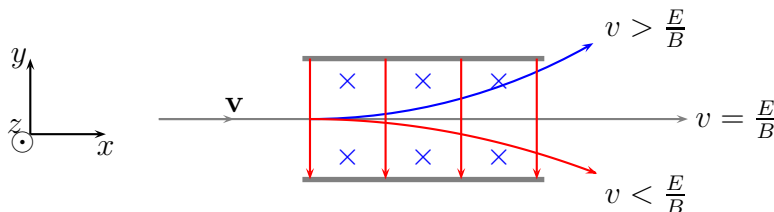


Figure 1: Problem 4

### Solution

5. (10 points.) You are driving in your car in the direction of positive  $x$ -axis with speed 31 m/s ( $\sim 70$  miles/hour). The magnetic field due to Earth in this region is in the  $xz$ -plane with its vertically downward component (along negative  $z$ -axis) having a magnitude of  $50 \mu\text{T}$ . The car being made of metal has charges that are free to move. These charges feel a magnetic force in the presence of the magnetic field and drift towards the sides of the car. Assuming the width of the car to be 1.0 m, determine the Hall voltage built up across the car.

**Solution**

6. (10 points.) A loop in the shape of a right triangle of sides  $a = 3.0 \text{ cm}$  and  $b = 2.0 \text{ cm}$ , carrying a current  $I = 2.0 \text{ A}$ , is placed in a magnetic field  $0.30 \text{ T}$  going into the page. See Figure 2. Determine the magnitude and direction of the force on side 3 of the triangle.

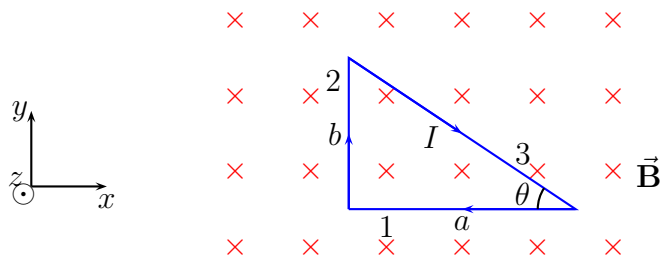


Figure 2: Problem 6.

**Solution**

7. (10 points.) A loop in the shape of a semi circle of radius  $R$ , carrying a current  $I$ , is placed in a magnetic field  $\mathbf{B}$ . See Figure 3. Determine the expression for magnitude and direction of the total force acting on the semi-circular part of the wire.

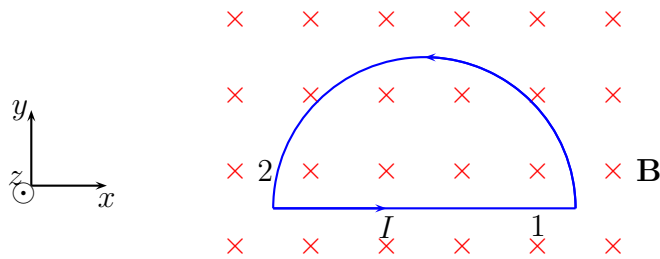


Figure 3: Problem 7.

**Solution**

8. (10 points.) A current of 16 mA is maintained in a single circular loop of  $1.20 \text{ m}^2$  area. A magnetic field of  $0.60 \text{ T}$  is directed parallel to the plane of the loop.
- (a) Calculate the magnetic dipole moment of the loop.

(b) What is the magnitude of the torque exerted by the magnetic field on the loop?

**Solution**