

Equation Sheet for PHYS-205A University Physics

(Last updated: January 12, 2024)

This list will evolve during the semester.

1. General mathematics:

(a) Units:

$$c = 10^{-2}, \quad m = 10^{-3}, \quad \mu = 10^{-6}, \quad n = 10^{-9}, \quad p = 10^{-12}. \quad (1a)$$

$$d = 10^2, \quad k = 10^3, \quad M = 10^6, \quad G = 10^9, \quad T = 10^{12}. \quad (1b)$$

(b) Geometry of a right triangle:

$$\sin \theta = \frac{\text{opp. to angle}}{\text{hypotenuse}}, \quad \tan \theta = \frac{\text{opp. to angle}}{\text{adj to angle}}, \quad (2a)$$

$$\cos \theta = \frac{\text{adj. to angle}}{\text{hypotenuse}}, \quad A^2 = A_x^2 + A_y^2. \quad (2b)$$

(c) Quadratic equation:

$$ax^2 + bx + c = 0, \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \quad (3)$$

(d) Calculus:

$$\frac{d}{dx}x^n = nx^{n-1}, \quad \int x^n dx = \frac{x^{n+1}}{n+1}. \quad (4)$$

2. Kinematic equations:

(a) Velocity and acceleration:

$$\mathbf{v} = \frac{d\mathbf{x}}{dt}, \quad \mathbf{a} = \frac{d\mathbf{v}}{dt}. \quad (5)$$

(b) Uniform velocity ($\mathbf{a} = 0$):

$$\Delta \mathbf{x} = \mathbf{v} \Delta t. \quad (6)$$

(c) Uniform acceleration:

$$v_f = v_i + a \Delta t; \quad \Delta x = v_i \Delta t + \frac{1}{2} a \Delta t^2; \quad v_f^2 = v_i^2 + 2 a \Delta x; \quad (7a)$$

$$\frac{\Delta x}{\Delta t} = \frac{v_i + v_f}{2}; \quad \Delta x = v_f \Delta t - \frac{1}{2} a \Delta t^2. \quad (7b)$$

(d) Circular motion:

$$f = \frac{1}{T}, \quad \omega = 2\pi f, \quad (8a)$$

$$v = \omega r = \frac{2\pi r}{T} = 2\pi f r, \quad (8b)$$

$$a_c = \frac{v^2}{r} = \omega^2 r = 4\pi^2 f^2 r = \frac{4\pi^2 r}{T^2} \quad (8c)$$

(e) Relative velocity: $\vec{\mathbf{v}}_{AB} = \vec{\mathbf{v}}_{AG} + \vec{\mathbf{v}}_{GB}$.

3. Forces:

(a) Newton's law:

$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \dots = m\vec{\mathbf{a}} \quad (9)$$

(b) Gravitational force:

$$F_G = \frac{Gm_1m_2}{R^2}, \quad G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2. \quad (10)$$

(c) Force due to friction:

$$F_f \begin{cases} \leq \mu_s N, & \text{(static case),} \\ = \mu_k N, & \text{(kinetic case).} \end{cases} \quad (11)$$

(d) Resistive forces: $R = bv$ (for small speeds) and $R = \frac{1}{2}D\rho Av^2$ (for high speeds).

4. Work and energy:

(a) Kinetic energy:

$$K = \frac{1}{2}mv^2 \quad (12)$$

(b) Work done by a force:

$$W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}} \rightarrow Fd \cos \theta \quad (13)$$

(c) Work-kinetic energy theorem:

$$W_1 + W_2 + \dots = \Delta K \quad (14)$$

(d) Potential energies:

$$U_g = mgh, \quad U_s = \frac{1}{2}kx^2. \quad (15)$$

(e) Mechanical energy:

$$\Delta K + \Delta U_g + \Delta U_s = W_{\text{fric}} + \dots \quad (16)$$

5. Linear momentum:

$$\vec{p} = m\vec{v}, \quad \vec{p}_f - \vec{p}_i = \int_i^f \vec{F} dt. \quad (17)$$

(a) Conservation of linear momentum:

$$\vec{P}_{1i} + \vec{P}_{2i} + \dots = \vec{P}_{1f} + \vec{P}_{2f} + \dots \quad (18)$$

(b) Elastic collision of two objects in one dimensional motion:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2} \right) v_{2i}, \quad (19a)$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2} \right) v_{2i}. \quad (19b)$$

(c) Center of mass:

$$X_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + \dots}{m_1 + m_2 + \dots} \rightarrow \frac{\int x dm}{\int dm}. \quad (20)$$

6. Rotational dynamics:

(a) Kinematic equations:

i. Constant angular speed ($\alpha = 0$): $\Delta\theta = \omega\Delta t$.

ii. Constant angular acceleration:

$$\omega_f = \omega_i + \alpha \Delta t, \quad \Delta\theta = \omega_i \Delta t + \frac{1}{2} \alpha \Delta t^2, \quad (21)$$

$$\omega_f^2 = \omega_i^2 + 2\alpha \Delta\theta, \quad \Delta\theta = \omega_f \Delta t - \frac{1}{2} \alpha \Delta t^2, \quad \frac{\Delta\theta}{\Delta t} = \frac{\omega_i + \omega_f}{2}. \quad (22)$$

(b) Rotational inertia (moment of inertia): $I = \int r^2 dm$.

$$I = \begin{cases} MR^2, & \text{Point mass, distance } R \text{ from axis,} \\ MR^2, & \text{Circular ring, about symmetry axis of ring,} \\ \frac{2}{3}MR^2, & \text{Spherical shell, about diameter,} \\ \frac{1}{2}MR^2, & \text{Solid cylinder, about symmetry axis of cylinder,} \\ \frac{2}{5}MR^2, & \text{Solid sphere, about diameter.} \end{cases} \quad (23a)$$

(c) Torque:

$$\tau = RF \sin \theta, \quad \vec{\tau} = \frac{d\vec{L}}{dt}. \quad (24)$$

(d) Rotational kinetic energy:

$$K_{\text{rot}} = \frac{1}{2} I \omega^2. \quad (25)$$

(e) Angular momentum:

$$L = I\omega, \quad \vec{L} = \vec{r} \times \vec{p}. \quad (26)$$

7. Gravitation:

$$\vec{F} = -\hat{r} \frac{Gm_1 m_2}{r^2}, \quad G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2. \quad (27)$$

$$U = -\frac{Gm_1 m_2}{r}. \quad (28)$$