Homework No. 06 (2022 Spring)

PHYS 520B: ELECTROMAGNETIC THEORY

Department of Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2022 Mar 22, 12.30pm

1. (**20 points.**) In terms of the four-vector potential

$$A^\mu = (\frac{1}{c}\phi, \mathbf{A}) \tag{1}$$

the Maxwell field tensor $F_{\mu\nu}$ is defined as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{2}$$

and the corresponding dual tensor is defined as

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}. \tag{3}$$

Derive the following relations, which involve quantities that remain invariant under Lorentz transformations.

(a) $c^2 F^{\mu\nu}F_{\mu\nu} = 2(c^2B^2 - E^2)$.

(b) $c^2 \tilde{F}^{\mu\nu}\tilde{F}_{\mu\nu} = 2(E^2 - c^2B^2)$.

(c) $cF^{\mu\nu}\tilde{F}_{\mu\nu} = -4\,\mathbf{B}\cdot\mathbf{E}$.