Homework No. 03 (2022 Spring)

PHYS 520B: ELECTROMAGNETIC THEORY

Department of Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2022 Feb 15, 4.30pm

1. (**20 points.**) A plane wave is incident, in vacuum, on a perfectly absorbing flat screen.

   (a) Without compromising generality we can choose the screen at $z = z_a$. Starting with the statement of conservation of linear momentum,
   $$\frac{\partial \mathbf{G}}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{T} + \mathbf{f} = 0, \tag{1}$$
   integrate on the volume between $z = z_a - \delta$ and $z = z_a + \delta$ for infinitely small $\delta > 0$. Interpret the integral of force density $\mathbf{f}$ as the total force, $\mathbf{F}$, on the plate. Further, note that the integral of momentum density $\mathbf{G}$ goes to zero for infinitely small δ. Thus, obtain
   $$\mathbf{F} = -\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{z_a-\delta}^{z_a+\delta} dz\, \boldsymbol{\nabla} \cdot \mathbf{T}. \tag{2}$$

   (b) Use divergence theorem to conclude
   $$\mathbf{F} = -\oint d\mathbf{a} \cdot \mathbf{T}, \tag{3}$$
   where the closed surface encloses the volume between $z = z_a - \delta$ and $z = z_a + \delta$ for infinitely small $\delta > 0$. Choose the plane wave to be incident on the side $z = z - \delta$ of the plate, and assuming $\mathbf{E} = 0$ and $\mathbf{B} = 0$ on the side $z = z + \delta$, conclude that
   $$\frac{\mathbf{F}}{A} = \hat{\mathbf{z}} \cdot \mathbf{T}\big|_{z=z_a-\delta}, \tag{4}$$
   where A is the total area of the screen. The electromagnetic stress tensor $\mathbf{T}$ in these expressions is given by
   $$\mathbf{T} = \mathbf{1}U - (\mathbf{DE} + \mathbf{BH}), \tag{5}$$
   where U is the electromagnetic energy density,
   $$U = \frac{1}{2}(\mathbf{D} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{H}). \tag{6}$$

   (c) For the particular case when the plane wave is incident normally on the screen ($\theta = 0$ in Fig. 1) calculate the force per unit area in the direction normal to the screen by evaluating
   $$\frac{\mathbf{F} \cdot \hat{\mathbf{z}}}{A}. \tag{7}$$
   Express the answer in terms of U using the properties of a plane wave: $\mathbf{k} \cdot \mathbf{E} = 0, \mathbf{k} \cdot \mathbf{B} = 0, \mathbf{E} \cdot \mathbf{B} = 0, |\mathbf{E}| = c|\mathbf{B}|$, and $kc = \omega$.

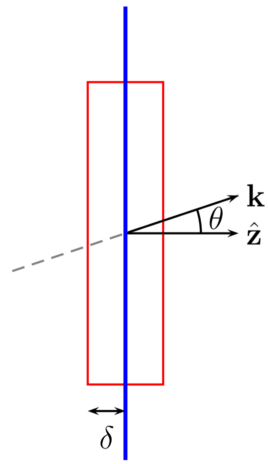

Figure 1: A plane wave with direction of propagation $\mathbf{k}$ incident on a screen.

(d) Consider the case when the plane wave is incident obliquely on the screen such that $\hat{\mathbf{k}} \cdot \hat{\mathbf{z}} = \cos\theta$ and $\mathbf{H} \cdot \hat{\mathbf{z}} = 0$. Calculate the force per unit area in the direction normal to the screen by evaluating

$$\frac{\mathbf{F} \cdot \hat{\mathbf{z}}}{A}, \tag{8}$$

and the force per unit area tangential to the screen by evaluating

$$\frac{\mathbf{F} \cdot \hat{\mathbf{x}}}{A}. \tag{9}$$

Express the answer in terms of U and θ using the properties of a plane wave.