Homework No. 01 (2022 Spring)

PHYS 520B: ELECTROMAGNETIC THEORY

Department of Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2022 Jan 18, 4.30pm

1. (**50 points.**) The Maxwell equations, in SI units, are

$$\boldsymbol{\nabla}\cdot\mathbf{D} = \rho, \tag{1a}$$

$$\boldsymbol{\nabla}\cdot\mathbf{B} = 0, \tag{1b}$$

$$-\boldsymbol{\nabla}\times\mathbf{E} - \frac{\partial}{\partial t}\mathbf{B} = 0, \tag{1c}$$

$$\boldsymbol{\nabla}\times\mathbf{H} - \frac{\partial}{\partial t}\mathbf{D} = \mathbf{J}, \tag{1d}$$

where

$$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}, \tag{2a}$$

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}. \tag{2b}$$

The Lorentz force, in SI units, is

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}. \tag{3}$$

We have

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}. \tag{4}$$

The above quantities will be addressed with subscripts SI in the following. The corresponding quantities in Gaussian (G) units and Heaviside-Lorentz (HL) units are obtained using the conversions

$$\sqrt{\frac{\varepsilon_0}{4\pi}}\,\mathbf{D}_{\text{G}} = \mathbf{D}_{\text{SI}} = \sqrt{\varepsilon_0}\,\mathbf{D}_{\text{HL}}, \qquad \sqrt{4\pi\varepsilon_0}\,\rho_{\text{G}} = \rho_{\text{SI}} = \sqrt{\varepsilon_0}\,\rho_{\text{HL}}, \tag{5a}$$

$$\frac{1}{\sqrt{4\pi\varepsilon_0}}\,\mathbf{E}_{\text{G}} = \mathbf{E}_{\text{SI}} = \frac{1}{\sqrt{\varepsilon_0}}\,\mathbf{E}_{\text{HL}}, \qquad \sqrt{4\pi\varepsilon_0}\,\mathbf{P}_{\text{G}} = \mathbf{P}_{\text{SI}} = \sqrt{\varepsilon_0}\,\mathbf{P}_{\text{HL}}, \tag{5b}$$

$$\frac{1}{\sqrt{4\pi\mu_0}}\,\mathbf{H}_{\text{G}} = \mathbf{H}_{\text{SI}} = \frac{1}{\sqrt{\mu_0}}\,\mathbf{H}_{\text{HL}}, \qquad \sqrt{4\pi\varepsilon_0}\,\mathbf{J}_{\text{G}} = \mathbf{J}_{\text{SI}} = \sqrt{\varepsilon_0}\,\mathbf{J}_{\text{HL}}, \tag{5c}$$

$$\sqrt{\frac{\mu_0}{4\pi}}\,\mathbf{B}_{\text{G}} = \mathbf{B}_{\text{SI}} = \sqrt{\mu_0}\,\mathbf{B}_{\text{HL}}, \qquad \sqrt{\frac{4\pi}{\mu_0}}\mathbf{M}_{\text{G}} = \mathbf{M}_{\text{SI}} = \frac{1}{\sqrt{\mu_0}}\mathbf{M}_{\text{HL}}. \tag{5d}$$

Note that the Heaviside-Lorentz units are obtained from Gaussian units by dropping the 4π's, which is called rationalization in this context.

(a) Starting from the Maxwell equations and Lorentz force in SI units, derive the corresponding equations in Gaussian units. The Maxwell equations, in Gaussian units, are

$$\begin{aligned} \boldsymbol{\nabla}\cdot\mathbf{D}_{\mathrm{G}} &= 4\pi\rho_{\mathrm{G}}, && (6a)\\ \boldsymbol{\nabla}\cdot\mathbf{B}_{\mathrm{G}} &= 0, && (6b)\\ -\boldsymbol{\nabla}\times\mathbf{E}_{\mathrm{G}} - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{B}_{\mathrm{G}} &= 0, && (6c)\\ \boldsymbol{\nabla}\times\mathbf{H}_{\mathrm{G}} - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{D}_{\mathrm{G}} &= \frac{4\pi}{c}\mathbf{J}_{\mathrm{G}}, && (6d)\end{aligned}$$

where

$$\begin{aligned} \mathbf{D}_{\mathrm{G}} &= \mathbf{E}_{\mathrm{G}} + 4\pi\mathbf{P}_{\mathrm{G}}, && (7a)\\ \mathbf{H}_{\mathrm{G}} &= \mathbf{B}_{\mathrm{G}} - 4\pi\mathbf{M}_{\mathrm{G}}. && (7b)\end{aligned}$$

The Lorentz force, in Gaussian units, is

$$\mathbf{F} = q_{\mathrm{G}}\mathbf{E}_{\mathrm{G}} + q_{\mathrm{G}}\frac{\mathbf{v}}{c}\times\mathbf{B}_{\mathrm{G}}. \tag{8}$$

Here charge q_{G} has the same conversion as charge density ρ_{G}.

(b) Starting from the Maxwell equations and Lorentz force in SI units, derive the corresponding equations in Lorentz-Heaviside units. The Maxwell equations, in Heaviside-Lorentz units, are

$$\begin{aligned} \boldsymbol{\nabla}\cdot\mathbf{D}_{\mathrm{HL}} &= \rho_{\mathrm{HL}}, && (9a)\\ \boldsymbol{\nabla}\cdot\mathbf{B}_{\mathrm{HL}} &= 0, && (9b)\\ -\boldsymbol{\nabla}\times\mathbf{E}_{\mathrm{HL}} - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{B}_{\mathrm{HL}} &= 0, && (9c)\\ \boldsymbol{\nabla}\times\mathbf{H}_{\mathrm{HL}} - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{D}_{\mathrm{HL}} &= \frac{1}{c}\mathbf{J}_{\mathrm{HL}}, && (9d)\end{aligned}$$

where

$$\begin{aligned} \mathbf{D}_{\mathrm{HL}} &= \mathbf{E}_{\mathrm{HL}} + \mathbf{P}_{\mathrm{HL}}, && (10a)\\ \mathbf{H}_{\mathrm{HL}} &= \mathbf{B}_{\mathrm{HL}} - \mathbf{M}_{\mathrm{HL}}. && (10b)\end{aligned}$$

The Lorentz force, in Heaviside-Lorentz units, is

$$\mathbf{F} = q_{\mathrm{HL}}\mathbf{E}_{\mathrm{HL}} + q_{\mathrm{HL}}\frac{\mathbf{v}}{c}\times\mathbf{B}_{\mathrm{HL}}. \tag{11}$$

Here charge q_{HL} has the same conversion as charge density ρ_{HL}.