Midterm Exam No. 01 (2022 Spring)

PHYS 520B: ELECTROMAGNETIC THEORY

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1. (**20 points.**) The electromagnetic energy density U and the corresponding energy flux vector $\mathbf{S}$ are given by, ($\mathbf{D} = \varepsilon_0 \mathbf{E}$, $\mathbf{B} = \mu_0 \mathbf{H}$, $\varepsilon_0 \mu_0 c^2 = 1$,)

$$U = \frac{1}{2}(\mathbf{D} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{H}), \qquad \mathbf{S} = \mathbf{E} \times \mathbf{H}. \tag{1}$$

The electromagnetic momentum density $\mathbf{G}$ and the corresponding momentum flux tensor $\mathbf{T}$ are given by

$$\mathbf{G} = \mathbf{D} \times \mathbf{B}, \qquad \mathbf{T} = \mathbf{1}U - (\mathbf{DE} + \mathbf{BH}). \tag{2}$$

Show that

$$\mathrm{Tr}(\mathbf{T}) = T_{ii} = aU, \tag{3}$$

where a is a number. Find a.

2. (**20 points.**) A monochromatic plane electromagnetic wave is described by electric and magnetic fields of the form

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}, \tag{4a}$$
$$\mathbf{B}(\mathbf{r}, t) = \mathbf{B}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}, \tag{4b}$$

where $\mathbf{E}_0$ and $\mathbf{B}_0$ are constants. Assume no charges or currents. Using Maxwell's equations show that

$$ck = \omega. \tag{5}$$

3. (**20 points.**) Let two infinitely thin plates occupying the $y = 0$ plane and $y = a$ olane consist of uniform charge density σ flowing in opposite directions in the x plane, described by drift velocity $\beta_d = v/c$ such that the electric and magnetic field for this configuration is given by

$$\mathbf{E} = \hat{\mathbf{y}}\, \frac{\sigma}{\varepsilon_0}, \quad 0 < y < a, \tag{6a}$$
$$c\mathbf{B} = \hat{\mathbf{z}}\, \beta_d E, \quad 0 < y < a. \tag{6b}$$

Thus, we have

$$cB = \beta_d E \tag{7}$$

in between the plates. Evaluate the force per unit area on the plate at $y = 0$.