Homework No. 05 (2022 Spring)

PHYS 510: CLASSICAL MECHANICS

Department of Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2022 Mar 15, 4.30pm

1. (**20 points.**) A system, characterized by the parameters ω, α, and β, and the dynamical parameter θ, is described by the equation of motion

$$\ddot{\theta} + \omega^2 \sin\theta + \alpha\ddot{\theta}\cos\theta + \beta\dot{\theta}^2 \sin\theta = 0. \tag{1}$$

Write the above equation of motion in the small angle approximation, to the leading order in θ.

2. (**20 points.**) Consider the coplanar double pendulum in Figure 2.

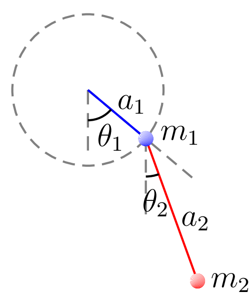

Figure 1: Problem 2.

(a) Write the Lagrangian for the system. in particular, show that the Lagrangian can be expressed in the form

$$L = L_1 + L_2 + L_{\text{int}}, \tag{2}$$

where

$$L_1 = \frac{1}{2}(m_1 + m_2)a_1^2\dot{\theta}_1^2 + (m_1 + m_2)ga_1\cos\theta_1, \tag{3a}$$

$$L_2 = \frac{1}{2}m_2 a_2^2\dot{\theta}_2^2 + m_2 g a_2 \cos\theta_2, \tag{3b}$$

$$L_{\text{int}} = m_2 a_1 a_2 \dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2). \tag{3c}$$

(b) Determine the equations of motion for the system. Express them in the form

$$(m_1 + m_2)a_1\ddot{\theta}_1 + (m_1 + m_2)g\sin\theta_1 + m_2 a_2\ddot{\theta}_2\cos(\theta_1 - \theta_2) + m_2 a_2\dot{\theta}_2^2\sin(\theta_1 - \theta_2) = 0, \tag{4a}$$

$$a_2\ddot{\theta}_2 + g\sin\theta_2 + a_1\ddot{\theta}_1\cos(\theta_1 - \theta_2) - a_1\dot{\theta}_1^2\sin(\theta_1 - \theta_2) = 0. \tag{4b}$$

(c) In the small angle approximation show that the equations of motion reduce to

$$\ddot{\theta}_1 + \omega_1^2 \theta_1 + \frac{\alpha}{\beta} \ddot{\theta}_2 = 0, \tag{5a}$$

$$\ddot{\theta}_2 + \omega_2^2 \theta_2 + \beta \ddot{\theta}_1 = 0, \tag{5b}$$

where

$$\omega_1^2 = \frac{g}{a_1}, \quad \omega_2^2 = \frac{g}{a_2}, \quad \alpha = \frac{m_2}{m_1 + m_2}, \quad \beta = \frac{a_1}{a_2} = \frac{\omega_2^2}{\omega_1^2}. \tag{6}$$

Note that $0 \leq \alpha \leq 1$.

(d) Determine the solution for the initial conditions

$$\theta_1(0) = 0, \quad \theta_2(0) = 0, \quad \dot{\theta}_1(0) = 0, \quad \dot{\theta}_2(0) = \omega_0, \tag{7}$$

for $\alpha = 1/2$ and $\beta = 1$.