

Homework No. 06 (Fall 2021)

PHYS 500A: MATHEMATICAL METHODS

Department of Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2021 Oct 5, 4.30pm

1. (20 points.) Evaluate the contour integral

$$I = \frac{1}{2\pi i} \oint_c dz \frac{e^{iz}}{(z^2 - a^2)}, \quad (1)$$

where the contour c is a unit circle going counterclockwise with center at the origin. Inquire the cases when $|a| > 1$ and $|a| < 1$.

2. (20 points.) Evaluate the integral

$$\int_{-\infty}^{\infty} \frac{dx e^{iax}}{x^2 + 1} \quad (2)$$

using Cauchy's theorem, after choosing a suitable contour. Here a is real.

3. (20 points.) Consider the integral

$$I(a) = \frac{1}{2\pi} \int_0^{2\pi} d\theta \frac{1}{1 - 2a \cos \theta + a^2}, \quad (3)$$

where a is complex.

- (a) Substitute $z = e^{i\theta}$, such that

$$2 \cos \theta = z + \frac{1}{z}, \quad (4)$$

and express the integral as a contour integral along the unit circle going counterclockwise. Locate the poles.

- (b) Evaluate the residues and show that

$$I(a) = \begin{cases} \frac{1}{1 - a^2}, & \text{if } |a| < 1, \\ \frac{1}{a^2 - 1}, & \text{if } |a| > 1. \end{cases} \quad (5)$$

- (c) Plot $I(a)$ for real values of a . Plot real and imaginary part of $I(a)$ for complex a . Argue that $I(1)$ is divergent.