

Homework No. 03 (Fall 2021)

PHYS 500A: MATHEMATICAL METHODS

Department of Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2021 Sep 14, 4.30pm

1. (10 points.) In term of unit vectors

$$\hat{\mathbf{r}} = \sin \theta \cos \phi \hat{\mathbf{i}} + \sin \theta \sin \phi \hat{\mathbf{j}} + \cos \theta \hat{\mathbf{k}}, \quad (1a)$$

$$\hat{\boldsymbol{\theta}} = \cos \theta \cos \phi \hat{\mathbf{i}} + \cos \theta \sin \phi \hat{\mathbf{j}} - \sin \theta \hat{\mathbf{k}}, \quad (1b)$$

$$\hat{\boldsymbol{\phi}} = -\sin \phi \hat{\mathbf{i}} + \cos \phi \hat{\mathbf{j}}, \quad (1c)$$

the basis vectors for spherical polar coordinates are

$$\mathbf{e}_r = \hat{\mathbf{r}} \quad \mathbf{e}^r = \hat{\mathbf{r}}, \quad (2a)$$

$$\mathbf{e}_\theta = r \hat{\boldsymbol{\theta}} \quad \mathbf{e}^\theta = \frac{1}{r} \hat{\boldsymbol{\theta}}, \quad (2b)$$

$$\mathbf{e}_\phi = r \sin \theta \hat{\boldsymbol{\phi}} \quad \mathbf{e}^\phi = \frac{1}{r \sin \theta} \hat{\boldsymbol{\phi}}. \quad (2c)$$

Compute the Christoffel symbols

$$\Gamma_{ij}^k = \left(\frac{\partial}{\partial u^j} \mathbf{e}_i \right) \cdot \mathbf{e}^k \quad (3)$$

for the spherical coordinate system.