Midterm Exam No. 02 (Fall 2021)

PHYS 500A: MATHEMATICAL METHODS

Department of Physics, Southern Illinois University–Carbondale
Date: 2021 Nov 4

1. (**20 points.**) Find the three roots that satisfy the equation

$$z^3 = -i. \tag{1}$$

   Mark the points corresponding to the three roots on the complex plane.

2. (**20 points.**) Evaluate

$$\frac{1}{2\pi i} \int_c \frac{dz}{z}, \tag{2}$$

   where the (open) contour c is along a semi circle of unit radius in the upper half plane going in the counterclockwise sense.

3. (**20 points.**) Evaluate the integral

$$\int_{-\infty}^{\infty} \frac{dx\, e^{iax}}{x^2 + k^2} \tag{3}$$

   using Cauchy's theorem, after choosing a suitable contour. Here a and k are real.

4. (**20 points.**) Find the matrix that diagonalizes

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{4}$$