Midterm Exam No. 01 (Fall 2021)

PHYS 500A: MATHEMATICAL METHODS

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1. (**20 points.**) Using the property of Kronecker δ-function and Levi-Civita symbol evaluate the following using index notation,

$$\varepsilon_{ijk}\varepsilon_{lmn}\delta_{il}\delta_{jm}\delta_{kn}. \tag{1}$$

2. (**20 points.**) Evaluate the left hand side of the equation

$$\boldsymbol{\nabla}(\mathbf{r}\cdot\mathbf{p})^2 = a\,\mathbf{p} + b\,\mathbf{r}, \tag{2}$$

where $\mathbf{p}$ is a constant vector. Thus, find a and b.

3. (**20 points.**) Given

$$\hat{\mathbf{r}} = \sin\theta\cos\phi\,\hat{\mathbf{i}} + \sin\theta\sin\phi\,\hat{\mathbf{j}} + \cos\theta\,\hat{\mathbf{k}}, \tag{3a}$$
$$\hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{i}} + \cos\theta\sin\phi\,\hat{\mathbf{j}} - \sin\theta\,\hat{\mathbf{k}}, \tag{3b}$$
$$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{i}} + \cos\phi\,\hat{\mathbf{j}}, \tag{3c}$$
$$\hat{\boldsymbol{\rho}} = \cos\phi\,\hat{\mathbf{i}} + \sin\phi\,\hat{\mathbf{j}}, \tag{3d}$$
$$\hat{\mathbf{z}} = \hat{\mathbf{k}}. \tag{3e}$$

and the relation

$$\hat{\boldsymbol{\theta}} = a\,\hat{\boldsymbol{\rho}} + b\,\hat{\boldsymbol{\phi}} + c\,\hat{\mathbf{z}}. \tag{4}$$

Find the components a, b, and c, such that the above equation is an identity.

4. (**20 points.**) Evaluate

$$\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{23}. \tag{5}$$

Mark the resulting number on the complex plane.

5. (**20 points.**) Find x and y in the relation

$$\tan^{-1}\left(\frac{3}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{y}{x}\right). \tag{6}$$