Homework No. 03 (Fall 2021)

PHYS 205B: University Physics

Due date: Thursday, 2021 Sep 16, 9:30 AM, on D2L

Instructions

- Describe your thought process in detail and organize it clearly. Make sure your answer has the correct units and the right number of significant digits.
- After completion, scan the pages as a single PDF file, and submit the file on D2L (under Assessments → Assignments).

Problems

1. (**10 points.**) A sphere with uniform charge distribution $-Q = -3.0\,\mu\text{C}$ is fixed at the origin. Point A is on a sphere of radius 5.0 cm and point B is on a sphere of radius 10.0 cm. Refer Figure 1.

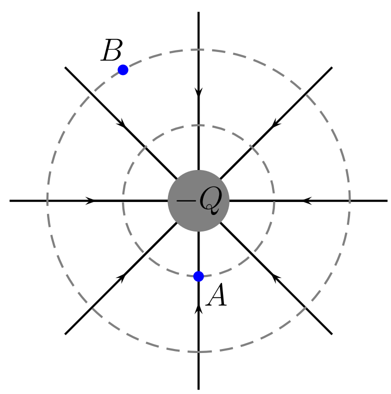

Figure 1: Problem 1

   (a) What is the work done by the electric force acting on charge $q = +2.0\,\mu\text{C}$, when q is moved from point A to point B.

   (b) What is the change in the electric potential energy between $-Q$ and q when q is moved from point A to point B.

   (c) If there are no other forces acting on charge q, using the work-energy theorem calculate the change in kinetic energy of charge q.

Solution

2. (**10 points.**) A cylinder with uniform line-charge distribution $\lambda = +5.0\,\mu\text{C/m}$ is fixed at the origin. Point A is on a cylinder of radius 10.0 cm and point B is on a cylinder of radius 20.0 cm. Refer Figure 2.

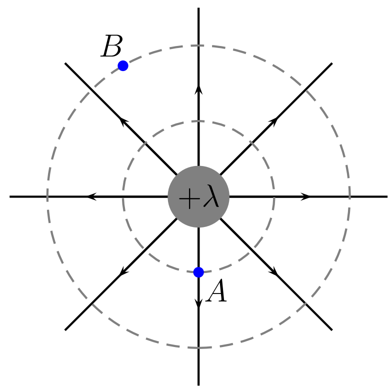

Figure 2: Problem 2

   (a) What is the work done by the electric force acting on charge $q = +2.0\,\mu\text{C}$, when q is moved from point A to point B.

   (b) What is the change in the electric potential energy between the line-charge λ and q when q is moved from point A to point B.

   (c) If there are no other forces acting on charge q, using the work-energy theorem calculate the change in kinetic energy of charge q.

Solution

3. (**10 points.**) A positive charge $Q_1 = 1.0\,\text{nC}$ is held fixed. Another positive charge $Q_2 = 2Q_1$ is tied to charge Q_1 using a string of length $a = 5.0\,\text{cm}$. Assume the radius of the two charges to be small in comparison to a. The charges have masses $m_1 = 0.05$ grams and $m_2 = 2m_1$. When the string is cut the two charges fly off in opposite directions. Determine the speed of each of the charges when they are (infinitely) far apart. (Hint: Use conservation of momentum and conservation of energy.)

Solution

4. (**10 points.**) Determine the total energy required to assemble four identical positive charges Q at the corners of a square of length L. Assume that the charges are brought from infinity.

Solution

5. (**10 points.**) Four charges $q_1 = q$, $q_2 = -2q$, $q_3 = -3q$, and $q_4 = 4q$, are placed at the corners of a square of side L, such that q_1 and q_4 are at diagonally opposite corners. Refer Figure 3.

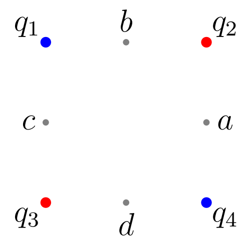

Figure 3: Problem 5

(a) What is the electric potential at the center of square?

(b) What is the electric potential at point a?

(c) What is the electric potential at point b?

(d) What is the electric potential difference between points a and c?

(e) How much potential energy is required to move another charge q from infinity to the center of the square?

(f) How much additional potential energy is required to move this charge from the center of the square to point a?

Solution

6. (**10 points.**) The electric dipole moment of a configuration consisting of two equal and opposite point charges, separated by a distance d, is defined to be

$$\mathbf{p} = q\mathbf{d}, \tag{1}$$

where $\mathbf{d}$ points from the negative to the positive charge and $d = |\mathbf{d}|$. Let $d = 2a$. The electric potential of the electric dipole at the point (x, y, z) is given by the expression

$$V(x,y,z) = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{(x^2+y^2+(z-a)^2}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{(x^2+y^2+(z+a)^2}}. \tag{2}$$

The electric field of the electric dipole can be calculated using

$$\mathbf{E}(x,y,z) = -\boldsymbol{\nabla} V \tag{3a}$$

$$= \hat{\mathbf{i}}\frac{\partial V}{\partial x} + \hat{\mathbf{j}}\frac{\partial V}{\partial y} + \hat{\mathbf{k}}\frac{\partial V}{\partial z} \tag{3b}$$

(a) Calculate the electric field on the x axis.

(b) Determine the electric field on the x axis for $a \ll x$.

(c) Calculate the electric field on the z axis.

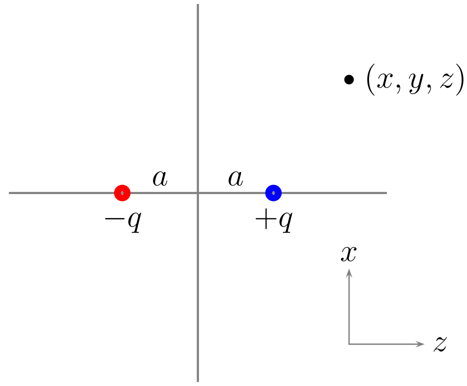

Figure 4: Problem 6

(d) Determine the electric field on the z axis for $a \ll z$.

Solution

7. (**10 points.**) Consider a uniformly charged disc of radius R with charge per unit area σ placed on the yz plane such that the origin is the center of the disc. Determine the electric potential on the x axis to be

$$V(x) = -\frac{\sigma}{2\varepsilon_0}\left[x - \sqrt{x^2 + R^2}\right]. \tag{4}$$

Using the fact that the electric field is the negative gradient of the electric potential, calculate the x-component of the electric field on the x axis to be

$$E_x = -\frac{\partial V}{\partial x} = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]. \tag{5}$$

(a) Show that this leads to the potential and electric field of a point charge in the limit $R \ll x$.

(b) Analyze the limit $x \ll R$. Show that this leads to the potential and electric field for a non-conducting uniformly charged plate. Plot the electric potential as a function of x for this case.

Solution

8. (**10 points.**) The electric field inside and outside a conducting sphere of radius R is given by

$$\mathbf{E} = \begin{cases} 0, & r < R \ \ \text{(inside)}, \\ \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}\hat{\mathbf{r}}, & R < r \ \ \text{(outside)}. \end{cases} \tag{6}$$

Determine the electric potential inside and outside the sphere using

$$\Delta V = -\int_{\mathbf{r}_i}^{\mathbf{r}_f} d\mathbf{l} \cdot \mathbf{E}. \tag{7}$$

Hint: Since the electric field is zero inside a perfect conductor, the electric potential inside the conductor must be a constant.

Solution