

# Homework No. 10 (Fall 2020)

## PHYS 500A: MATHEMATICAL METHODS

*Department of Physics, Southern Illinois University–Carbondale*

Due date: Thursday, 2020 Dec 3, 9.30am

1. **(20 points.)** Generate 3D plots of surface spherical harmonics  $Y_{lm}(\theta, \phi)$  as a function of  $\theta$  and  $\phi$ . In particular,
  - (a) Plot  $\text{Re}[Y_{73}(\theta, \phi)]$ .
  - (b) Plot  $\text{Im}[Y_{73}(\theta, \phi)]$ .
  - (c) Plot  $\text{Abs}[Y_{73}(\theta, \phi)]$ .
  - (d) Plot your favourite spherical harmonic, that is, choose a  $l$  and  $m$ , and Re or Im or Abs.

Hint: In Mathematica these plots are generated using the following commands:

```
SphericalPlot3D[Re[SphericalHarmonicY[1,m,θ,φ]],{θ,0,Pi},{φ,0,2 Pi}]
```

```
SphericalPlot3D[Im[SphericalHarmonicY[1,m,θ,φ]],{θ,0,Pi},{φ,0,2 Pi}]
```

```
SphericalPlot3D[Abs[SphericalHarmonicY[1,m,θ,φ]],{θ,0,Pi},{φ,0,2 Pi}]
```

Refer to diagrams in Wikipedia article on ‘spherical harmonics’ to see some visual representations of these functions.

2. **(20 points.)** The spherical harmonics are given by

$$Y_{lm}(\theta, \phi) = \sqrt{\frac{2l+1}{4\pi}} \sqrt{\frac{(l+m)!}{(l-m)!}} \left(\frac{e^{i\phi}}{\sin\theta}\right)^m \left(\frac{d}{d\cos\theta}\right)^{l-m} \frac{(\cos^2\theta - 1)^l}{2^l l!}. \quad (1)$$

Express  $Y_{lm}(\theta, \phi)$  in terms of  $l$ ,  $\phi$  and  $\sin\theta$ .