Homework No. 07 (Fall 2020)

PHYS 500A: MATHEMATICAL METHODS

Department of Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2020 Oct 27, 9.30am

1. (**20 points.**) Show that

$$\oint_{c1} dz \ln z = 2\pi i R, \tag{1a}$$

$$\oint_{c2} dz \ln z = 0, \tag{1b}$$

where the contours $c1$ and $c2$ are shown in Figure 1, and R is the radius of the circle forming the contour. Is the function $\ln z$ analytic at $z = 0$? Is the function $\ln z$ analytic at $z \neq 0$? Show that if the contour c winds around the origin more than once the integral evaluates to

$$\oint_{c} dz \ln z = 2\pi i R n, \tag{2}$$

where n is the number of times the contour winds around the origin.
Hint: Show that

$$\oint_{c1} dz \ln z = -R \int_0^{2\pi} \theta d\theta e^{i\theta}. \tag{3}$$

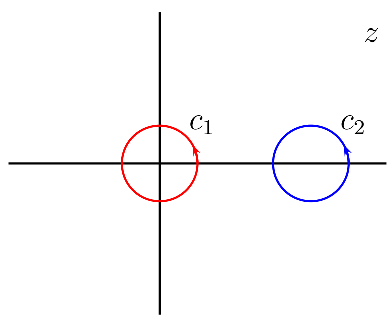

Figure 1: Contour c_1 encircles the origin while contour c_2 does not encircle the origin.

2. (**20 points.**) Evaluate the integral

$$I(\lambda) = \frac{1}{\pi} \int_0^{\infty} \frac{x^{\frac{1}{3}}\, dx}{1 + 2x\cos\lambda + x^2}. \tag{4}$$