Homework No. 03 (Fall 2020)

PHYS 500A: MATHEMATICAL METHODS

Department of Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2020 Sep 8, 9.30am

1. (**10 points.**) In term of unit vectors

$$\hat{\mathbf{r}} = \sin\theta\cos\phi\,\hat{\mathbf{i}} + \sin\theta\sin\phi\,\hat{\mathbf{j}} + \cos\theta\,\hat{\mathbf{k}}, \tag{1a}$$

$$\hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{i}} + \cos\theta\sin\phi\,\hat{\mathbf{j}} - \sin\theta\,\hat{\mathbf{k}}, \tag{1b}$$

$$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{i}} + \cos\phi\,\hat{\mathbf{j}}, \tag{1c}$$

the basis vectors for spherical polar coordinates are

$$\mathbf{e}_r = \hat{\mathbf{r}} \qquad \mathbf{e}^r = \hat{\mathbf{r}}, \tag{2a}$$

$$\mathbf{e}_\theta = r\hat{\boldsymbol{\theta}} \qquad \mathbf{e}^\theta = \frac{1}{r}\hat{\boldsymbol{\theta}}, \tag{2b}$$

$$\mathbf{e}_\phi = r\sin\theta\hat{\boldsymbol{\phi}} \qquad \mathbf{e}^\phi = \frac{1}{r\sin\theta}\hat{\boldsymbol{\phi}}. \tag{2c}$$

Compute the Christoffel symbols

$$\Gamma^k_{ij} = \left(\frac{\partial}{\partial u^j}\mathbf{e}_i\right)\cdot\mathbf{e}^k \tag{3}$$

for the spherical coordinate system.