Homework No. 01 (Fall 2020)

PHYS 500A: MATHEMATICAL METHODS

Department of Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2020 Aug 25, 9.30am

1. (**10 points.**) In spherical polar coordinates a point is coordinated by the intersection of family of spheres, cones, and half-planes, given by

$$r = \sqrt{x^2 + y^2 + z^2}, \tag{1a}$$

$$\theta = \tan^{-1} \sqrt{\frac{x^2 + y^2}{z^2}}, \tag{1b}$$

$$\phi = \tan^{-1} \frac{y}{x}, \tag{1c}$$

respectively. Show that the gradient of these surfaces are given by

$$\boldsymbol{\nabla} r = \hat{\mathbf{r}}, \qquad \hat{\mathbf{r}} = \sin\theta\cos\phi\,\hat{\mathbf{i}} + \sin\theta\sin\phi\,\hat{\mathbf{j}} + \cos\theta\,\hat{\mathbf{k}}, \tag{2a}$$

$$\boldsymbol{\nabla}\theta = \hat{\boldsymbol{\theta}}\,\frac{1}{r}, \qquad \hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{i}} + \cos\theta\sin\phi\,\hat{\mathbf{j}} - \sin\theta\,\hat{\mathbf{k}}, \tag{2b}$$

$$\boldsymbol{\nabla}\phi = \hat{\boldsymbol{\phi}}\,\frac{1}{r\sin\theta}, \qquad \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{i}} + \cos\phi\,\hat{\mathbf{j}}, \tag{2c}$$

which are normal to the respective surfaces. Sketch the surfaces and the corresponding normal vectors. This illustrates that $\boldsymbol{\nabla}$(surface) is a vector (field) normal to the surface.