Midterm Exam No. 02 (Fall 2020)

PHYS 500A: MATHEMATICAL METHODS

Department of Physics, Southern Illinois University–Carbondale

Date: 2020 Oct 9

1. (**20 points.**) Locate $z = \pi^i$ on the complex plane.

2. (**20 points.**) Find all the roots the equation

$$z^3 = -1. \tag{1}$$

   Mark the points corresponding to the roots on the complex plane.

3. (**20 points.**) Check if the function

$$f(z) = \frac{1}{z} \tag{2}$$

   satisfies the Cauchy-Riemann conditions.

   (a) Verify that the Cauchy-Riemann conditions for this case are not well defined at $z = 0$, but are fine for $z \neq 0$.

   (b) Verify that

$$\frac{df}{dz} = -\frac{1}{z^2}, \qquad z \neq 0. \tag{3}$$

   (c) Determine the limiting value of the derivative as you approach $z = 0$ along the positive real line, and, then, when you approach along the negative real line. Repeat the analysis along the imaginary line. Repeat the analysis along the line $x = y$. Are these limits identical?

   (d) If these limits are not identical conclude that the derivative is not isotropic at $z = 0$. Then, the function is not analytic at $z = 0$.

4. (**20 points.**) Evaluate $\boldsymbol{\nabla} \times \mathbf{A}$, given

$$\mathbf{A}(\mathbf{r}) = \hat{\boldsymbol{\phi}} \frac{(1 - \cos\theta)}{r \sin\theta}. \tag{4}$$

5. (**20 points.**) Show that the complex function

$$f = zz^* \tag{5}$$

   is not an analytic function. Express the function f in the form $f = u + iv$ and find that $u = (x^2 + y^2)$ and $v = 0$. Show that the Cauchy-Riemann conditions are not satisfied in this case. Could we modify the function v, keeping u the same, such that the new function f is analytic? That is, investigate if there exists a function v that satisfies the Cauchy-Riemann conditions with $u = (x^2 + y^2)$? If yes, find a v, and interpret? If no, what is the interpretation?