Homework No. 12 (2020 Fall)

PHYS 320: ELECTRICITY AND MAGNETISM I

Due date: Friday, 2020 Dec 4, 2:00 PM, on D2L

1. (**20 points.**) A monochromatic plane electromagnetic wave is described by electric and magnetic fields of the form

$$\mathbf{E} = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}, \tag{1a}$$
$$\mathbf{B} = \mathbf{B}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}, \tag{1b}$$

where $\mathbf{E}_0$ and $\mathbf{B}_0$ are constants. Assume no charges or currents.

(a) Using Maxwell's equations show that

$$\mathbf{k}\cdot\mathbf{E} = 0, \tag{2a}$$
$$\mathbf{k}\cdot\mathbf{B} = 0, \tag{2b}$$
$$\mathbf{k}\times\mathbf{E} = \omega\mathbf{B}, \tag{2c}$$
$$\mathbf{k}\times\mathbf{B} = -\frac{\omega}{c^2}\mathbf{E}, \tag{2d}$$

where $\varepsilon_0\mu_0 = 1/c^2$.

(b) For non-trivial cases ($\mathbf{E}_0 \neq 0$ and $\mathbf{B}_0 \neq 0$), using Eqs. (2), show that we have

$$ck = \omega. \tag{3}$$

Then, deduce the relations

$$\mathbf{E}^*\cdot\mathbf{B} = 0, \tag{4}$$
$$\mathbf{E}^*\times\mathbf{B} = \hat{\mathbf{k}}\frac{1}{c}|\mathbf{E}|^2 = \hat{\mathbf{k}}c|\mathbf{B}|^2. \tag{5}$$

Thus, we have

$$E = cB. \tag{6}$$

(c) Evaluate the electromagnetic energy density

$$U = \frac{1}{2}\mathbf{D}^*\cdot\mathbf{E} + \frac{1}{2}\mathbf{B}^*\cdot\mathbf{H} \tag{7}$$

and the electromagnetic momentum density

$$\mathbf{G} = \mathbf{D}^*\times\mathbf{B}. \tag{8}$$

Then, determine the ratio U/G. What is the interpretation?