Homework No. 05 (2020 Fall)

PHYS 320: Electricity and Magnetism I

Due date: Friday, 2020 Sep 11, 2:00 PM, on D2L

1. (**20 points.**) Using Gauss's law find the electric field inside and outside a solid sphere of radius R with total charge Q distributed inside the sphere with a charge density

$$\rho(\mathbf{r}) = br\,\theta(R - r), \tag{1}$$

where r is the distance from the center of sphere. Here $\theta(x) = 1$, if $x > 0$, and 0 otherwise.

2. (**20 points.**) Using Gauss's law find the electric field in a region, a distance R away from the origin, if the charge density in space is given

$$\rho(\mathbf{r}) = \frac{\sigma}{r}, \tag{2}$$

where r is the radial distance from origin and σ is a parameter with units of charge per unit area.

3. (**20 points.**) (Problem 2.15 Griffiths 4th/3rd edition.)
A thick spherical shell carries charge density

$$\rho(\mathbf{r}) = \frac{k}{r^2}, \quad a \le r \le b. \tag{3}$$

Find the electric field in the three regions: (i) $r < a$, (ii) $a < r < b$, (iii) $b < r$. Plot $|\mathbf{E}|$ as a function of r, for the case $b = 2a$.

4. (**40 points.**) Consider a uniformly charged solid sphere of radius R with total charge Q.

   (a) Using Gauss's law show that the electric field inside and outside the sphere is given by

$$\mathbf{E}(\mathbf{r}) = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0}\dfrac{1}{R^2}\dfrac{r}{R}\,\hat{\mathbf{r}}, & r < R, \\ \dfrac{Q}{4\pi\varepsilon_0}\dfrac{1}{r^2}\,\hat{\mathbf{r}}, & r > R, \end{cases} \tag{4}$$

   where $\mathbf{r}$ is the radial vector with respect to the center of sphere.

   (b) Plot the magnitude of the electric field as a function of r.

   (c) Rewrite your results for the case when the solid sphere is a perfect conductor?

   (d) Rewrite your results for the case of a uniformly charged hollow sphere of radius R with total charge Q.