Homework No. 03 (2020 Fall)

PHYS 320: Electricity and Magnetism I

Due date: None

1. (**10 points.**) (Based on problem 1.26 Griffiths 4th edition.)
   Calculate the Laplacian of the function
   $$T_a = x^2 + 2xy + 3z + 4. \tag{1}$$

2. (**10 points.**) (Based on problem 1.32/1.31 Griffiths 4th/3rd edition.)
   Check the fundamental theorem for gradients,
   $$\int_{\mathbf{a}}^{\mathbf{b}} d\mathbf{l} \cdot \boldsymbol{\nabla} T = T(\mathbf{b}) - T(\mathbf{a}), \tag{2}$$
   using $T = x^2 + 4xy + 2yz^3$, the points $\mathbf{a} = (0,0,0)$, $\mathbf{b} = (1,1,1)$, integrated along the path in Fig. 1.28 (a) in Griffiths, obtained by connecting the points
   $$(0,0,0) \to (1,0,0) \to (1,1,0) \to (1,1,1). \tag{3}$$

3. (**10 points.**) (Based on problem 1.33/1.32 Griffiths 4th/3rd edition.)
   Check the fundamental theorem of divergence,
   $$\int_V d^3x\, \boldsymbol{\nabla} \cdot \mathbf{E} = \oint_S d\mathbf{a} \cdot \mathbf{E}, \tag{4}$$
   for the vector field $\mathbf{E} = x\,\hat{\mathbf{x}}$. Take a cube of length L as your volume, which is placed with one edge parallel to the x-axis. Using the fact that the divergence of a vector field at a point tells us whether a point is a source or sink of the field, estimate the distribution of the source and sink for the field $\mathbf{E}$?

4. (**10 points.**) (Based on problem 1.34/1.33 Griffiths 4th/3rd edition.)
   Check the fundamental theorem of curl,
   $$\int_S d\mathbf{a} \cdot \boldsymbol{\nabla} \times \mathbf{E} = \oint_C d\mathbf{l} \cdot \mathbf{E}, \tag{5}$$
   (where the sense of the line integration is given by the right hand rule: the contour C is traversed in the sense of the fingers of the right hand and the thumb points in the sense of the orientation of the surface,) for the vector field $\mathbf{E} = y\,\hat{\mathbf{x}} + z\,\hat{\mathbf{y}} + x\,\hat{\mathbf{z}}$. Take a square of length L on the $z = 0$ plane as your surface, which is placed with one side parallel to the x-axis. Using the fact that the curl of a vector field at a point is a measure of the torque experienced by a (point) dipole at the point, estimate the torque field.