Homework No. 02 (2020 Fall)

PHYS 320: Electricity and Magnetism I

Due date: Monday, 2020 Aug 31, 2:00 PM, in class or on D2L

1. (**20 points.**) Verify the following identities:

$$\boldsymbol{\nabla} r = \frac{\mathbf{r}}{r} = \hat{\mathbf{r}}, \tag{1a}$$

$$\boldsymbol{\nabla}\, \mathbf{r} = \mathbf{1}. \tag{1b}$$

   Further, show that

$$\boldsymbol{\nabla} \cdot \mathbf{r} = 3, \tag{2a}$$

$$\boldsymbol{\nabla} \times \mathbf{r} = 0. \tag{2b}$$

   Here r is the magnitude of the position vector $\mathbf{r}$, and $\hat{\mathbf{r}}$ is the unit vector pointing in the direction of $\mathbf{r}$.

2. (**20 points.**) (Based on Problem 1.13, Griffiths 4th edition.)
   Show that

$$\boldsymbol{\nabla} r^2 = 2\mathbf{r}. \tag{3}$$

   Then evaluate $\boldsymbol{\nabla} r^3$. Show that

$$\boldsymbol{\nabla} \frac{1}{r} = -\frac{\hat{\mathbf{r}}}{r^2}. \tag{4}$$

   Then evaluate

$$\boldsymbol{\nabla} \left(\frac{1}{r^2} \right). \tag{5}$$

3. (**20 points.**) Use index notation or dyadic notation to show that

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{A}) = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{A}) - \nabla^2 \mathbf{A}, \tag{6a}$$

$$\boldsymbol{\nabla} \cdot (\mathbf{A} \times \mathbf{B}) = (\boldsymbol{\nabla} \times \mathbf{A}) \cdot \mathbf{B} - \mathbf{A} \cdot (\boldsymbol{\nabla} \times \mathbf{B}). \tag{6b}$$