Midterm Exam No. 02 (Fall 2020)

PHYS 320: ELECTRICITY AND MAGNETISM I

Department of Physics, Southern Illinois University–Carbondale

Date: 2020 Oct 9

1. (**20 points.**) Three identical point charges, each of charge q, are placed at the corners of an equilateral triangle of side a.

   (a) Determine the electric potential at the center of the triangle.

   (b) How much potential energy is required to move another proton from infinity to the center of the triangle?

2. (**20 points.**) Two electrons and two protons are placed at the corners of a square of length a, such that the electrons are at diagonally opposite corners. For simplicity let us choose them to be in the xy plane. Find the monopole moment, the dipole moment, and the quadrupole moment, of this configuration of four charges.

3. (**20 points.**) The Legendre polynomials of order l satisfy the recurrence relation

$$(2l+1)xP_l(x) = (l+1)P_{l+1}(x) + lP_{l-1}(x), \qquad l = 1, 2, 3, \ldots. \tag{1}$$

   Recall,

$$P_0(x) = 1, \tag{2a}$$
$$P_1(x) = x. \tag{2b}$$

   Derive the explicit expression for $P_4(x)$ using the recurrence relation.

4. (**20 points.**) The surface charge density on the surface of a charged sphere is given by

$$\sigma(\theta) = \frac{Q}{4\pi a^2}\cos^2\theta, \tag{3}$$

   where θ is the polar angle in spherical coordinates. Express this charge distribution in terms of the Legendre polynomials. Recall,

$$P_0(\cos\theta) = 1, \tag{4a}$$
$$P_1(\cos\theta) = \cos\theta, \tag{4b}$$
$$P_2(\cos\theta) = \frac{3}{2}\cos^2\theta - \frac{1}{2}. \tag{4c}$$

   Determine the electric potential *on the z-axis* due to this charge distribution.