Final Exam (Fall 2020)

PHYS 320: ELECTRICITY AND MAGNETISM I

Department of Physics, Southern Illinois University–Carbondale

Due date: Friday, 2020 Dec 11, 4:45 PM, on D2L

1. (**20 points.**) Evaluate
$$\nabla^2(\mathbf{r}\cdot\mathbf{a})^2, \tag{1}$$
given $\mathbf{a}$ is a constant vector.

2. (**20 points.**) Evaluate the integral
$$\int_{-1}^{1} dx\,\delta(1-2x)\Big[8x^2+2x-1\Big]. \tag{2}$$
(Caution: Be careful to avoid a possible error in sign.)

3. (**20 points.**) Consider a uniformly charged solid sphere of radius R with total charge Q.

   (a) Using Gauss's law show that the electric field inside and outside the sphere is given by
$$\mathbf{E}(\mathbf{r}) = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0}\dfrac{\mathbf{r}}{R^3}, & r<R, \\ \dfrac{Q}{4\pi\varepsilon_0}\dfrac{\mathbf{r}}{r^3}, & r>R, \end{cases} \tag{3}$$
   where $\mathbf{r}$ is the radial vector with respect to the center of sphere.

   (b) Plot the magnitude of the electric field as a function of r.

4. (**20 points.**) The Legendre polynomials of order l satisfy the recurrence relation
$$(2l+1)xP_l(x) = (l+1)P_{l+1}(x) + lP_{l-1}(x), \qquad l=1,2,3,\ldots. \tag{4}$$
Recall,
$$P_0(x) = 1, \tag{5a}$$
$$P_1(x) = x. \tag{5b}$$
Derive the explicit expression for $P_3(x)$ using the recurrence relation.

5. (**20 points.**) Determine the magnitude and direction of the electrostatic force on a positive charge placed a distance a away from a grounded perfectly conducting plate.