Midterm Exam No. 01 (Spring 2017)

PHYS 510: Classical Mechanics

Date: 2017 Feb 28

1. (**20 points.**) Give an account of the functional derivative

$$\frac{\delta f(x)}{\delta f(x')} = \delta(x - x'). \tag{1}$$

2. (**20 points.**) Given

$$F[u] = \int_{x_1}^{x_2} dx\, a(x) \frac{du(x)}{dx}. \tag{2}$$

Evaluate

$$\frac{\delta F}{\delta u(x)}. \tag{3}$$

3. (**20 points.**) Find the geodesics on the surface of a circular cylinder. Identify these curves.
Hint: To have a visual perception of these geodesics it helps to note that a cylinder can be mapped (or cut open) into a plane.

4. (**20 points.**) Consider the coplanar double pendulum in Figure 1. In the small angle

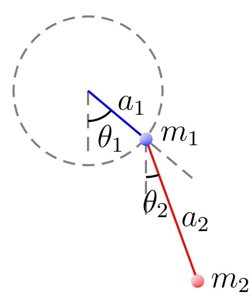

Figure 1: Problem 4.

approximation the equations of motion for double pendulum reduce to

$$\ddot{\theta}_1 + \omega_1^2 \theta_1 + \frac{\alpha}{\beta} \ddot{\theta}_2 = 0, \tag{4a}$$

$$\ddot{\theta}_2 + \omega_2^2 \theta_1 + \beta \ddot{\theta}_1 = 0, \tag{4b}$$

where

$$\omega_1^2 = \frac{g}{a_1}, \quad \omega_2^2 = \frac{g}{a_2}, \quad \alpha = \frac{m_2}{m_1 + m_2}, \quad \beta = \frac{a_1}{a_2} = \frac{\omega_2^2}{\omega_1^2}. \tag{5}$$

Note that $0 \leq \alpha \leq 1$. Determine the solution for the initial conditions

$$\theta_1(0) = 0, \quad \theta_2(0) = 0, \quad \dot{\theta}_1(0) = 0, \quad \dot{\theta}_2(0) = \omega_0, \tag{6}$$

for $\alpha = 1/2$ and $\beta = 1$.