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Green's function for radiation

1. $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ — (i)

$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \Rightarrow \quad \vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$ — (ii)

$\vec{\nabla} \cdot \vec{B} = 0 \quad \Rightarrow \quad \vec{B} = \vec{\nabla} \times \vec{A}$ — (iii)

$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$ — (iv)

2. Using (ii) and (iii) in (i) and (iv) we have the following two equations describing electrodynamics,

$$-\nabla^2 \phi = \frac{\rho}{\epsilon_0} + \frac{\partial}{\partial t}(\vec{\nabla} \cdot \vec{A})$$

$$\left(-\nabla^2 + \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2}\right) \vec{A} = \mu_0 \vec{J} - \vec{\nabla}\left(\vec{\nabla} \cdot \vec{A} + \mu_0 \epsilon_0 \frac{\partial \phi}{\partial t}\right)$$

3. <u>Radiation gauge</u> (also called Coulomb gauge) → used by Maxwell

$$\boxed{\vec{\nabla} \cdot \vec{A} = 0}$$

$$-\nabla^2 \phi = \frac{\rho}{\epsilon_0}$$

$$\left(-\nabla^2 + \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2}\right) \vec{A} = \mu_0 \vec{J} - \mu_0 \epsilon_0 \frac{\partial}{\partial t} \vec{\nabla}\phi$$

4. <u>Lorenz gauge</u>

$$\boxed{\vec{\nabla} \cdot \vec{A} + \mu_0 \epsilon_0 \frac{\partial \phi}{\partial t} = 0}$$

→ introduced by Lorenz, not Lorentz, in 1850's.
→ Was not common in use untill Hertz's experiment in late 1880's.

$$\left(-\nabla^2 + \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2}\right) \phi = \frac{\rho}{\epsilon_0}$$

$$\left(-\nabla^2 + \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2}\right) \vec{A} = \mu_0 \vec{J}$$

5 If one chooses to not introduce potentials, we have,

$$\left(-\nabla^2 + \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2}\right) \vec{E} = -\vec{\nabla} \frac{\rho}{\epsilon_0} - \epsilon_0 \mu_0 \frac{\partial \vec{J}}{\partial t}$$ ← HW.

$$\left(-\nabla^2 + \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2}\right) \vec{B} = \mu_0 \vec{\nabla} \times \vec{J}$$

This is derived from (ii) and (iv) of 1 using the vector identity, $\vec{\nabla} \times (\vec{\nabla} \times \vec{X}) = +\vec{\nabla}(\vec{\nabla} \cdot \vec{X}) - \nabla^2 \vec{X}$.

6 The implication of 5 is that the fields $\vec{E}$ and $\vec{B}$ satisfy the wave equation in regions outside the sources ρ and $\vec{J}$. In particular the wave has its speed given in terms of the property of vacuum, ϵ_0 and μ_0,

$$c^2 = \frac{1}{\epsilon_0 \mu_0}.$$

This motivated the concept of ether, and eventually to the Special Theory of Relativity.