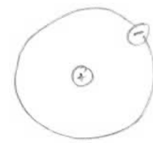
SET - 01

1 Classical model of atom

$$F = \frac{ke^2}{r^2} \qquad a = \frac{v^2}{r} \qquad \Rightarrow \quad mv^2 = \frac{ke^2}{r}$$

$$ma = \frac{ke^2}{r^2}$$



$$\text{Energy} = E = K + V$$
$$= \frac{1}{2}mv^2 - \frac{ke^2}{r}$$
$$= -\frac{1}{2}\frac{ke^2}{r}$$

2 Power radiated by an accelerating charge. (Larmor formula)

$$P = \frac{2}{3}\frac{ke^2}{c^3}a^2$$

Thus, for an electron orbiting inside the atom

$$P = \frac{2}{3}\frac{ke^2}{c^3}\left(\frac{ke^2}{mr^2}\right)^2$$

3 Using 1

$$P = -\frac{dE}{dt} = \frac{d}{dt}\frac{1}{2}\frac{ke^2}{r}$$
$$= -\frac{1}{2}\frac{ke^2}{r^2}\frac{dr}{dt}$$

4 Comparing 2 and 3 we have.

$$-\frac{1}{2}\frac{\cancel{ke^2}}{r^2}\frac{dr}{dt} = \frac{2}{3}\frac{\cancel{ke^2}}{c^3}\left(\frac{ke^2}{m}\right)^2\frac{1}{r^4}$$

$$-r^2 dr = \frac{4}{3}c\,dt\left(\frac{ke^2}{mc^2}\right)^2$$

5 $\frac{ke^2}{mc^2} = r_0$ — classical radius of electron.

$\sim 3 \times 10^{-15}$ m.

6 $-r^2 dr = \frac{4}{3} c\,dt\, r_0^2$.

$$-\int_R^0 r^2 dr = \frac{4}{3} r_0^2 \int_0^T c\,dt$$

R – initial radius
$\sim$ Bohr radius $\sim 10^{-10}$ m.
T – classical life time.

$$\frac{R^3}{\cancel{3}} = \frac{4}{\cancel{3}} r_0^2 cT$$

$$T = \frac{R^3}{4 r_0^2 c} = \frac{(10^{-10}\,m)^3}{(3 \times 10^{-15} m)^2 (3 \times 10^8 \frac{m}{s})\, 4}$$

$$= \frac{10^{-30}}{10 \times 10^{-30}\ 10^9} \sim 10^{-10}\ \text{sec.}$$

7 Some Atoms like H, Li have lifetimes more than the universe.

$\sim 13.8 \times 10^9$ y $\sim 10^{17}$ sec.

(8) Heuristic explanation for stability of atoms

$$E = \frac{1}{2} m v^2 - \frac{ke^2}{r}$$

$$= \frac{1}{2m} p^2 - \frac{ke^2}{r}$$

(9) Uncertainty principle

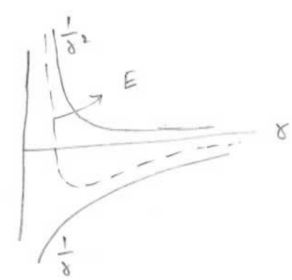
$$\Delta r \, \Delta p_r \gtrsim \hbar$$

let the uncertainty in r and p_r be of the order in the atoms, thus,

$$p \sim \frac{\hbar}{r}$$

(10) Using (8) and (9)

$$E = \frac{\hbar^2}{2m} \frac{1}{r^2} - \frac{ke^2}{r}$$



(11) What is E_{min}?

$$\frac{\partial E}{\partial r} = -2 \frac{\hbar^2}{2m} \frac{1}{r^3} + \frac{ke^2}{r^2} = 0$$

$$2 \frac{\hbar^2}{2m} \frac{1}{r_{min}^2} = \frac{ke^2}{r_{min}}$$

$$r_{min} = 2 \frac{\hbar^2}{2m(ke^2)}$$

$$E_{min} = -\frac{1}{2} \frac{ke^2}{r_{min}} = -\frac{1}{2} \frac{m(ke^2)^2}{\hbar^2}$$

(12) Bohr model.

$$mvr = n\hbar$$

$$mv = n\frac{\hbar}{r}$$

$$E = \frac{p^2}{2m} - \frac{ke^2}{r}$$

$$= \frac{n^2\hbar^2}{2m}\frac{1}{r^2} - \frac{ke^2}{r}$$

Replacing $\hbar \to n\hbar$ in (11)

$$r_{min} = \frac{n^2\hbar^2}{2m(ke^2)}$$

$$E_{min} = -\frac{1}{2}\frac{m(ke^2)^2}{n^2\hbar^2}$$

(13) Improvement of Bohr model by taking care of the motion of proton

$$m \to \mu = \frac{mM}{m+M} = \frac{m}{\left(1+\frac{m}{M}\right)} \qquad \frac{m}{M} \sim \frac{1}{2000}$$

$$\sim m\left[1 - \frac{m}{M}\right]$$

(14) Frank-Hertz experiment.