SET - 07

Dielectric models and response functions

① $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ (defining $\vec{D} = \epsilon \vec{E}$)

$$\epsilon \vec{E} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{P} = \epsilon_0 \left(\frac{\epsilon}{\epsilon_0} - 1 \right) \vec{E}$$

$$= \epsilon_0 \chi \vec{E}$$ (defining $\chi = \frac{\epsilon}{\epsilon_0} - 1$)

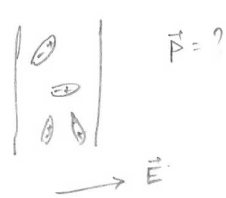
ϵ - diedectric permmitivitty

χ - electric susceptibility

② In general

$$\vec{P} = \epsilon_0 \overleftrightarrow{\chi} \cdot \vec{E}$$

③ What are the physical requirements for polarization to be a response to an applied electric field



④ Physical requirements:

(i) Response is local

(ii) Response is causal

(iii) Translation invariance in time

(iv) $\chi(t)$ should be real.

(5) $$\vec{P}(\vec{x},t) = \epsilon_0 \int d^3x' \int dt' \; \chi(\vec{x},\vec{x}',t,t') \; \vec{E}(\vec{x}',t')$$

$\downarrow$ local

$$\vec{P}(\vec{x},t) = \epsilon_0 \int_{-\infty}^{+\infty} dt' \; \chi(\vec{x},t,t') \; \vec{E}(\vec{x},t')$$

$\downarrow$ causal

$$\vec{P}(\vec{x},t) = \epsilon_0 \int_{-\infty}^{t} dt' \; \chi(\vec{x},t,t') \; \vec{E}(\vec{x},t')$$

$\downarrow$ translational invariance.

$$\vec{P}(\vec{x},t) = \epsilon_0 \int_{-\infty}^{t} dt' \; \chi(\vec{x},t-t') \; \vec{E}(\vec{x},t')$$

(6) We shall stop writing the $\vec{x}$ dependence.

$$\vec{P}(t) = \epsilon_0 \int_{-\infty}^{t} dt' \; \chi(t-t') \; \vec{E}(t')$$

(7) The causal relation is explicitly stated as

$$\chi(t-t') = \theta(t-t') \; f(t-t'),$$

where
$$\theta(t) = \begin{cases} 1 & \text{if } t>0 \\ 0 & \text{if } t<0 \end{cases}$$

(8) $$\vec{P}(t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{-i\omega t}\, \vec{P}(\omega)$$

$$\vec{P}(\omega) = \int_{-\infty}^{+\infty} dt\, e^{i\omega t}\, \vec{P}(t)$$

(9) $$\vec{P}(\omega) = \epsilon_0 \int_{-\infty}^{+\infty} dt\, e^{i\omega t} \int_{-\infty}^{+\infty} dt'\, \chi(t-t')\, \vec{E}(t')$$

$$= \epsilon_0 \int_{-\infty}^{+\infty} dt\, e^{i\omega t} \int_{-\infty}^{+\infty} dt' \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi}\, e^{-i\omega'(t-t')}\, \chi(\omega') \int_{-\infty}^{+\infty} \frac{d\omega''}{2\pi}\, e^{-i\omega'' t'}\, \vec{E}(\omega'')$$

$$= \epsilon_0 \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \int_{-\infty}^{+\infty} \frac{d\omega''}{2\pi}\, \chi(\omega')\, \vec{E}(\omega'') \int_{-\infty}^{+\infty} dt\, e^{it(\omega-\omega')} \int_{-\infty}^{+\infty} dt'\, e^{it'(\omega'-\omega'')}$$

$$= \epsilon_0 \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \int_{-\infty}^{+\infty} \frac{d\omega''}{2\pi}\, \chi(\omega')\, \vec{E}(\omega'')\; 2\pi\, \delta(\omega-\omega')\; 2\pi\, \delta(\omega'-\omega'')$$

$$\vec{P}(\omega) = \epsilon_0\, \chi(\omega)\, \vec{E}(\omega)$$

(10) The response should be real.

$$\chi^*(t) = \chi(t)$$

$$\left[\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{-i\omega t}\, \chi(\omega) \right]^* = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{-i\omega t}\, \chi(\omega)$$

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{i\omega t}\, \chi^*(\omega) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{-i\omega t}\, \chi(\omega)$$

$$\omega \to -\omega$$

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{-i\omega t}\, \chi^*(-\omega) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{-i\omega t}\, \chi(\omega)$$

$$\Rightarrow \quad \chi^*(-\omega) = \chi(\omega).$$

(11) Let us consider a model for the response function for a material consisting of bound and free charges.

$$m \vec{a} = \vec{F}_{net}$$

$$m \frac{d^2 \vec{x}(t)}{dt^2} = e \vec{E}(t) - m \omega_0^2 \vec{x}(t) - m \gamma \frac{d \vec{x}(t)}{dt}$$

for bound charges.

represents collisions and dissipation

(12) $$\vec{x}(t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \vec{x}(\omega)$$

$$\frac{d}{dt} \rightarrow -i\omega$$

$$m (-i\omega)^2 \vec{x}(\omega) = e \vec{E}(\omega) - m \omega_0^2 \vec{x}(\omega) - m \gamma (-i\omega) \vec{x}(\omega)$$

$$\left[\omega_0^2 - \omega^2 - i\omega\gamma \right] \vec{x}(\omega) = \frac{e}{m} \vec{E}(\omega)$$

$$\vec{x}(\omega) = \frac{\frac{e}{m}}{\left[\omega_0^2 - \omega^2 - i\omega\gamma \right]} \vec{E}(\omega)$$

(13) Using

$$\vec{P} = n_b e \vec{x}$$

we have

$$\vec{P}(\omega) = \epsilon_0 \frac{\left(\frac{n_b e^2}{m \epsilon_0}\right)}{\left[\omega_0^2 - \omega^2 - i\omega\Gamma\right]} \vec{E}(\omega)$$

which leads to the recognition

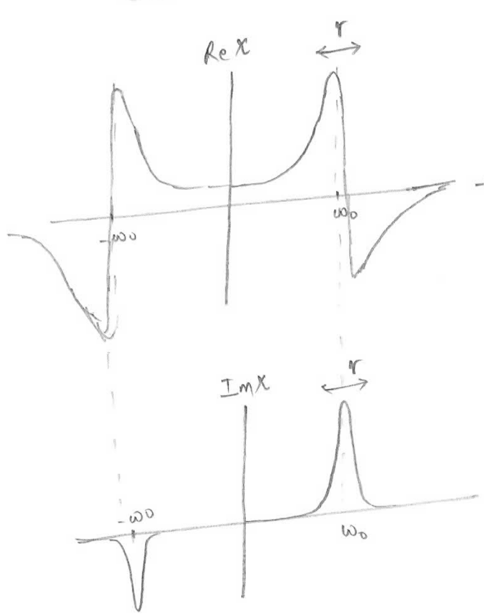
$$\omega_b^2 = \frac{n_b e^2}{m \epsilon_0}$$

$$\chi(\omega) = \frac{\omega_b^2}{\omega_0^2 - \omega^2 - i\omega\Gamma}$$

(14)
$$\chi(\omega) = \frac{\omega_b^2\left[(\omega_0^2 - \omega^2) + i\omega\Gamma\right]}{(\omega_0^2-\omega^2)^2 + \omega^2\Gamma^2}$$

→ fluctuation-dissipation theorem
→ Johnson noise
→ Nyquist, Callen.

Insulator ($\Gamma << \omega_0$)



Conductor ($\omega_0 << \Gamma$)