Homework No. 08 (Fall 2013)

PHYS 520A: Electromagnetic Theory I

Due date: Wednesday, 2013 Nov 20, 4.30pm

1. Find the solution to the differential equation

$$\left[-\frac{\partial}{\partial z}\varepsilon(z)\frac{\partial}{\partial z}+\varepsilon(z)k_\perp^2\right]g(z,z';k_\perp)=\delta(z-z') \tag{1}$$

when

$$\varepsilon(z)=\begin{cases}\varepsilon_2 & z<a,\\ \varepsilon_1 & a<z.\end{cases} \tag{2}$$

for the case $a<z'$. Look for solutions that is zero at $z=\pm\infty$.

2. Consider a semi-infinite dielectric slab described by

$$\varepsilon(z)=\begin{cases}\varepsilon_2 & z<a,\\ \varepsilon_1>\varepsilon_2 & a<z.\end{cases} \tag{3}$$

   (a) Find the expression for the electric field due to a point charge q placed at $\mathbf{r}'$ (with $a<z'$).

   (b) Investigate the continuity in the components of electric field found above at the interface by evaluating the following:

$$E_x(x,y,a+\delta)-E_x(x,y,a-\delta)=?, \tag{4}$$
$$E_y(x,y,a+\delta)-E_y(x,y,a-\delta)=?, \tag{5}$$
$$\varepsilon_1E_z(x,y,a+\delta)-\varepsilon_2E_z(x,y,a-\delta)=?. \tag{6}$$

3. Consider a semi-infinite dielectric slab described by

$$\varepsilon(z)=\begin{cases}\varepsilon_2 & z<a,\\ \varepsilon_1>\varepsilon_2 & a<z.\end{cases} \tag{7}$$

Find the expression for the electric potential due to a point dipole $\mathbf{d}$ placed at $\mathbf{r}'$ (with $a<z'$).
Hint: The charge density for a point dipole is

$$\rho(\mathbf{r})=-\mathbf{d}\cdot\boldsymbol{\nabla}\delta^{(3)}(\mathbf{r}-\mathbf{r}'). \tag{8}$$