Homework No. 05 (Fall 2013)

PHYS 520A: Electromagnetic Theory I

Due date: Wednesday, 2013 Oct 30, 4.30pm

1. Evaluate the principal value of the integral, ($\delta > 0$,)

$$\int_{-\infty}^{\infty} \frac{dx}{(x+i\delta)}. \tag{1}$$

2. Show that the energy density in a dispersive medium is given by

$$U(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi}\, e^{-i(\omega'-\omega)t}\, U(\omega,\omega'), \tag{2}$$

where

$$U(\omega,\omega') = \frac{1}{2}\mathbf{E}(-\omega)\cdot \frac{\big[\omega'\boldsymbol{\varepsilon}(\omega') - \omega\boldsymbol{\varepsilon}(\omega)\big]}{\omega'-\omega}\cdot\mathbf{E}(\omega') + \frac{1}{2}\mathbf{H}(-\omega)\cdot \frac{\big[\omega'\boldsymbol{\mu}(\omega') - \omega\boldsymbol{\mu}(\omega)\big]}{\omega'-\omega}\cdot\mathbf{H}(\omega'). \tag{3}$$

3. Show that the speed of energy flow of a monochromatic electromagnetic wave in a dispersive medium (for slowly evolving field) when both ε and μ are frequency dependent is given by

$$\frac{v_E}{c} = \left[\frac{d}{d\omega}\left(\omega\sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}\right)\right]^{-1}. \tag{4}$$

Determine the speed of energy flow for the case

$$\mu = \mu_0 \qquad \text{and} \qquad \frac{\varepsilon}{\varepsilon_0} = 1 - \frac{\omega_p^2}{\omega^2} \tag{5}$$

to be

$$\frac{v_E}{c} = \sqrt{1-\frac{\omega_p^2}{\omega^2}} < 1. \tag{6}$$