Exam No. 02 (Fall 2013)

PHYS 520A: Electromagnetic Theory I

Date: 2013 Oct 24

1. Show that the effective charge density, ρ_{eff}, and the effective current density, $\mathbf{j}_{\text{eff}}$,

$$\rho_{\text{eff}} = -\boldsymbol{\nabla} \cdot \mathbf{P}, \tag{1}$$

$$\mathbf{j}_{\text{eff}} = \frac{\partial}{\partial t}\mathbf{P} + \boldsymbol{\nabla} \times \mathbf{M}, \tag{2}$$

satisfy the equation of charge conservation

$$\frac{\partial}{\partial t}\rho_{\text{eff}} + \boldsymbol{\nabla} \cdot \mathbf{j}_{\text{eff}} = 0. \tag{3}$$

2. Consider the charge density

$$\rho(\mathbf{r}) = -\mathbf{d} \cdot \boldsymbol{\nabla}\delta^{(3)}(\mathbf{r}). \tag{4}$$

(a) Find the total charge of the charge density by evaluating

$$\int d^3r\, \rho(\mathbf{r}). \tag{5}$$

(b) Find the dipole moment of the charge density by evaluating

$$\int d^3r\, \mathbf{r}\, \rho(\mathbf{r}). \tag{6}$$

3. The response to an electric field in the Drude model is described by the susceptibility function

$$\chi(\omega) = \frac{\omega_p^2}{\omega_0^2 - i\omega\gamma}. \tag{7}$$

Plot $[\text{Re}\chi(\omega)]$ as a function of ω.

4. Consider a circular loop of wire carrying current I whose magnetic moment is given by $\boldsymbol{\mu} = IA\hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ points perpendicular to the plane containing the loop (satisfying the right hand sense) and A is the area of the loop. Consider the case $\hat{\mathbf{n}} = \hat{\mathbf{x}}$. What is the magnitude and direction of the torque experienced by this loop in the presence of a uniform magnetic field $\mathbf{B} = B\hat{\mathbf{y}}$. Describe the resultant motion of the loop. (Hint: The torque experienced by a magnetic moment $\boldsymbol{\mu}$ in a magnetic field $\mathbf{B}$ is $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$.)

5. A simple model for susceptibility is

$$\chi(\omega) = \frac{\omega_1}{\omega_0 - \omega} + i\,\pi\omega_1\delta(\omega - \omega_0), \tag{8}$$

where ω_0 and ω_1 represent physical parameters of a material.

(a) Note that

$$[\mathrm{Re}\chi(\omega)] = \frac{\omega_1}{\omega_0 - \omega} \qquad \text{and} \qquad [\mathrm{Im}\chi(\omega)] = \pi\omega_1\delta(\omega - \omega_0). \tag{9}$$

(b) Plot $[\mathrm{Re}\chi(\omega)]$ and $[\mathrm{Im}\chi(\omega)]$ with respect to ω.

(c) Evaluate the right hand side of the Kramers-Kronig relation

$$[\mathrm{Re}\chi(\omega)] = \lim_{\delta\to 0+}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}[\mathrm{Im}\chi(\omega')]\,2\mathrm{Re}\left\{\frac{1}{\omega' - (\omega + i\delta)}\right\} \tag{10}$$

for this simple model.